Holography and Related Topics in String Theory

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Abstract

This thesis covers research into the holographic entropy bound, the D-brane descent relations, and properties of AdS_2 solutions in supersymmetric and nonsupersymmetric string theories. The first chapter introduces these topics and the connections between them.

In chapter two, the holographic Bousso bound is studied and its modification due to quantum effects is presented. The Bousso bound requires that one quarter the area of a closed codimension two spacelike surface exceeds the entropy flux across a certain lightsheet terminating on the surface. The bound can be violated by quantum effects such as Hawking radiation. It is proposed that, at the quantum level, the bound be modified by adding to the area the quantum entanglement entropy across the surface. The validity of this quantum Bousso bound is proven in a two-dimensional large-N dilaton gravity theory.

On the topic of D-branes, chapter three introduces the descent relations among branes of different dimensionality and stability. The descent relations require special attention in the nonsupersymmetric type-0 theories which have twice as many stable D-branes as the type-II theories. In light of this added complication, the descent relations in the type 0A and 0B theories, as well as the D-branes' couplings to NS-NS fields, are worked out in detail in this chapter.

With an eye to AdS_2/CFT_1 holography, the first of two studies of AdS_2 is begun in chapter four. In the context of two-dimensional type-0A string theory, a family of AdS solutions to the effective action are presented. This family of solutions may be parameterized by two independent variables: the tachyon expectation value and the

string coupling constant.

Chapter five turns attention to a possible supersymmetric arena for $\mathrm{AdS}_2/\mathrm{CFT}_1$ duality, namely $AdS_2 \times S^2 \times CY_3$ flux compactifications of type IIA string theory. The problem of finding supersymmetric brane configurations in the near-horizon attractor geometry of a Calabi-Yau black hole with magnetic-electric charges (p^I, q_I) is considered. Half-BPS configurations, which are static for some choice of global AdS_2 coordinate, are found for wrapped brane configurations with essentially any four-dimensional charges (u^I, v_I). Half-BPS multibrane configurations can also be found for any collection of wrapped branes provided they all have the same sign for the symplectic inner product $p^I v_I - u^I q_I$ of their charges with the black hole charges. This contrasts with the Minkowski problem for which a mutually preserved supersymmetry requires alignment of all the charge vectors. The radial position of the branes in global AdS_2 is determined by the phase of their central charge.

Contents

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Chapter 1

Introduction

Since the discovery of black hole thermodynamics [1–3], physicists have suspected that gravitational theories obey what is known as the holographic principle [4, 5]. Despite the appellation of "principle," holography is actually an unproven, yet well-motivated, conjecture about the behavior of theories with gravity. The holographic principle claims that theories of gravity can be described entirely by a non-gravitational theory on a codimension-one boundary of spacetime. In particular, the three spatial dimensions of our universe may be a holographic illusion generated by some as-yet-unknown physics in 2+1 dimensions.

The first piece of evidence for holography came from studies of black holes, when it was found that the entropy of a black hole scales as the area of the horizon, $S_{bh} = A/4$. The creation of these black holes places a natural limit on the amount of information that can be crammed into a volume V. Once an amount of matter is confined to a region smaller than its Schwarzschild radius, gravity ensures that the system becomes hidden behind a horizon. The original matter system can have no more entropy than the resulting black hole, that is, no more entropy than one-quarter its area. This contradicts conventional wisdom that the number of degrees of freedom in a quantum field theory should scale as the volume, corresponding to the number of available Planck volumes. Instead, we learn that, like black holes, the maximum entropy in any region of space must scale as the area. In this thesis, we report progress in understanding this holographic principle and related topics in string theory. The bulk of this thesis has been reported in references [6–9].

In the case of a black hole, the horizon provides a natural surface to associate with the volume of the black hole. In more general spacetimes, or for arbitrary regions of space, it is less obvious how to choose a bounding area. Bousso has proposed a general prescription for associating areas with volumes [10]. This prescription, known as the "Bousso bound," has been proven in classical regimes under a variety of assumptions [11, 12], but, as shown in chapter 2, the bound can fail once quantum effects are taken into account. We show that the area is corrected by the entanglement entropy and that this "quantum Bousso bound" holds in a two-dimensional large-N model, thus salvaging the holographic principle in the quantum regime [7].

The AdS/CFT correspondence is perhaps the best understood example of a holographic system. The degrees of freedom for the gravity system (AdS) can be projected onto the dual theory (CFT) living on the boundary of spacetime. This duality was first discovered by studying the near-horizon geometry of coincident D3-branes [13]. The gravitational physics in the vicinity of the D-branes is dual to the non-gravitational physics on the D-branes. This correspondence motivates a closer look at D-branes and, in particular, we can ask how the D-branes localize and how D-branes of different dimensions are related to each other. Sen discovered an amazing web of relationships among D-branes in the type-IIA and IIB theories which goes a long way towards answering these questions [14]. In chapter 3, we investigate the nature of these so-called "descent relations" in the type-0A and 0B theories [6].

Theories of gravity in two dimensions are generally simpler to study analytically than theories of gravity in higher dimensions. For this reason, studies of $\mathrm{AdS}_2/\mathrm{CFT}_1$ hold great promise for furthering our understanding of holography. Towards this end, in chapter 4, we present a family of AdS solutions in two-dimensional type-0A string theory [8]. In chapter 5, we study the supersymmetric configurations of D-branes in $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_3$ flux compactifications of type-IIA string theory [9].

Chapter 2

A Quantum Bousso Bound

2.1 Introduction

The generalized second law of thermodynamics (GSL) [1–3] states that one quarter the area of black hole horizons plus the entropy outside the horizons is nondecreasing. This law was formulated in an attempt to repair inconsistencies in the ordinary second law in the presence of black holes. There is no precise general statement, let alone proof, of the GSL, but it has been demonstrated in a compelling variety of special circumstances. It indicates a deep connection between geometry, thermodynamics and quantum mechanics which we have yet to fathom. The holographic principle [4, 5], which also has no precise general statement, endeavors to elevate and extend the GSL to contexts not necessarily involving black holes. In [10], a mathematically precise modification of the GSL/holographic principle was proposed that is applicable to null surfaces which are *not* horizons [15]. This proposed "Bousso bound", along with a generalization stated therein, was proven, subject to certain conditions, in a classical limit by Flanagan, Marolf, and Wald [11].

The Bousso bound, as stated, can be violated by quantum effects [16]. Mathematically, the proofs of the bound rely on the local positivity of the stress tensor which does not hold in the quantum world. Physically, the bound does not account for entropy carried by Hawking radiation. In this chapter, we propose that, at a semiclassical level, the bound can be restored by adding to one quarter the surface area the entanglement entropy across the surface. We will make this statement fully precise, and then prove it, in a two-dimensional model of large N dilaton gravity.

This chapter is organized as follows. We begin by reviewing Bousso's covariant entropy bound in section 2.2. We will review the lightsheet construction in general D-dimensional spacetime, although our main interest in the remainder of the chapter will be four and two dimensions. In section 2.3, we will discuss how Bousso's bound can be violated in the presence of semiclassical effects, like Hawking radiation. This will motivate us to propose a "quantum Bousso bound" in section 2.4. By assuming an adiabaticity condition on the entropy flux, we will show in section 2.5 that the classical Bousso bound can be proven in four and two dimensions. In section 2.6, we extend the analysis to the two-dimensional RST quantization [17, 18] of the CGHS model [19] which includes semiclassical Hawking radiation and its backreaction. We will show that the quantum Bousso bound holds in this gravitational theory.

2.2 Review of the classical Bousso bounds

The Bousso bound asserts that, subject to certain assumptions, the entropy of matter that passes through certain lightsheets associated with a given codimension-two spatial surface in spacetime is bounded by the area of that surface [10].

This entropy bound provides a covariant recipe for associating a geometric entropy with any spatial surface B that is codimension two in the spacetime. At each point of B, there are four null directions orthogonal to B. These four null directions single out four unique null geodesics emanating from each point of B: two future-directed and two past-directed. Without loss of generality, we choose an affine parameter λ on each of these curves such that λ equals zero on B and increases positively as the geodesic is followed away from B.

Along each of the four geodesics, labelled by i, an expansion parameter $\theta_i(\lambda) = \nabla_a \left(\frac{d}{d\lambda}\right)^a$ can be defined. If we note that each of the future-directed geodesics is simply the extension of one of the past-directed geodesics, then the following relations between the expansion parameters becomes clear: $\theta_1(0) = -\theta_3(0)$, $\theta_2(0) = -\theta_4(0)$. Therefore, at least two of the four geodesics will begin with a nonpositive expansion. A "lightsheet" is a codimension-one surface generated by following exactly one non-expanding geodesic from each point of B. Each geodesic is followed until one of the following occurs on it:

- The expansion parameter becomes positive, $\theta > 0$,
- A spacetime singularity is reached.

Note that, in spacetime dimensions greater than two, there are an infinite number of possible lightsheets to choose from since, for each point on B, there are at least two contracting null geodesics from which to choose.

The original Bousso bound conjectures that Nature obeys the following inequality:

$$\textbf{Entropy passing through any lightsheet of } B \leq \frac{1}{4}(\textbf{ Area of } B)\ . \quad (2.2.1)$$

In order to make this statement precise, we must clarify what we mean by the entropy that passes through a lightsheet. In general, this is ambiguous because entropy is not a local concept. However, there is a thermodynamic limit in which the entropy is well-approximated by the flux of a four-vector s^a. As discussed by Flanagan, Marolf, and Wald (FMW) in [11], this thermodynamic limit is satisfied under the entropy condition that we will use in sections 2.5 and 2.6. The Bousso bound as formulated so far pertains mainly to this limit.

To find the entropy flux that passes through the lightsheet, we must project s^a onto k^b, the unique future-directed normal to the lightsheet. Up to a sign, k is $d/d\lambda$ since $d/d\lambda$ is null and orthogonal to all other lightsheet tangent vectors by construction. In order to keep k^a future-directed, we choose $k^a = \left(\frac{d}{d\lambda}\right)^a$ if the lightsheet is future-directed, and $k^a = -\left(\frac{d}{d\lambda}\right)^a$ if the lightsheet is past-directed. Since we use the mostly-positive metric signature, the entropy flux through any point of the lightsheet is

$$s \equiv -k_a s^a \,. \tag{2.2.2}$$

In the language of entropy flux, the entropy bound becomes

$$\int_{L(B)} s \;\leq\; \frac{1}{4}(\text{Area of } B)\,, \tag{2.2.3}$$

where $L(B)$ denotes the lightsheet of B. However, there is a generalized Bousso bound [11] in which the lightsheet is prematurely terminated on a spatial surface B'. It is clear that the integral of s over this terminated lightsheet equals the integral over the full lightsheet of B minus the integral over the full lightsheet of B'. Assuming that s is everywhere positive, Bousso's original entropy bound tells us that

$$\int_{L(B-B')} s \;\leq\; \int_{L(B)} s \;\leq\; \frac{1}{4}(\text{Area of } B)\,, \tag{2.2.4}$$

where $L(B-B')$ denotes the lightsheet of B terminated on B'. In this chapter, we will be interested in the generalized Bousso bound, first proposed by FMW [11], which imposes the much stronger bound on the terminated lightsheet:

$$\int_{L(B-B')} s \;\leq\; \frac{1}{4}\left(A(B) - A(B')\right)\,. \tag{2.2.5}$$

This has been proven under suitable assumptions by FMW [11]. Note that this generalized entropy bound directly implies Bousso's original entropy bound.

2.3 Semiclassical violations

The entropy bounds so far pertain largely to the classical regime. When quantum effects are included, even at the semiclassical level, we expect that the bounds must be somehow modified to account for the entropy carried by Hawking radiation. Mathematically, the proofs [11] are not applicable because quantum effects violate the positive energy condition.

The classical proofs hinge on the focussing theorem of classical general relativity. The focussing theorem, in turn, derives from the Raychaudhuri equation and the null energy condition. The Raychaudhuri equation provides a differential equation for the expansion parameter along a null geodesic [20]:

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - 8\pi T_{ab}k^a k^b \, , \tag{2.3.1}$$

where σ_{ab} is the shear tensor and ω_{ab} is the twist tensor. For a family of null geodesics that start off orthogonal to a spatial surface, such as the case for a lightsheet, the twist tensor is zero. Finally, if we assume that the null energy condition holds, then the last term is negative. The null energy condition postulates that $T_{ab}k^a k^b$ is nonnegative for all null vectors k^a. As a result, we find that the expansion parameter satisfies the inequality

$$\frac{d\theta}{d\lambda} \leq -\frac{1}{D-2}\theta^2 \, . \tag{2.3.2}$$

This gives us the focussing theorem: If the expansion parameter takes the negative value θ_0 along a null geodesic of the lightsheet, then that geodesic will reach a caustic (i.e., $\theta \to -\infty$) within the finite affine time $\Delta\lambda \leq \frac{D-2}{|\theta_0|}$.

So long as energy is required to produce entropy, the focussing theorem ensures that the presence of entropy will cause the lightsheet to reach a caustic and, therefore, terminate. The more entropy we try to pass through the lightsheet, the faster the lightsheet terminates. This gives a compelling argument for why only a finite, bounded amount of entropy could be passed through the lightsheet. According to the Bousso bound, this upper bound is precisely one quarter the area of the generating surface.

In practice, the covariant entropy bound can be violated in the presence of matter with negative energy. By mixing positive-energy matter and negative-energy matter, a system with zero energy can be made to carry an arbitrary amount of entropy. Again, the entropy passing through any given lightsheet could be increased arbitrarily. At the classical level, we could simply demand that the energy-momentum tensor obeys the null energy energy condition. This is the weakest of all the most common energy conditions and, as can be seen from (2.3.1), is the one needed for the focussing theorem, and thus to make the Bousso bound plausible.

However, the Bousso bound is in serious trouble once we include quantum effects. We know that none of the local energy conditions can hold even at first order in $\hbar$. In particular, the phenomenon of Hawking radiation violates the null energy condition near the horizon of black holes. This allows for violations of the focussing theorem. This violation can be seen most clearly for future-directed, outgoing null geodesics that hover for a while in between the event horizon and apparent horizon of an evaporating black hole. The apparent horizon is the boundary of the region of trapped surfaces, so the congruence of null geodesics are contracting inside the apparent horizon. However, as the black hole evaporates, the apparent horizon follows a timelike trajectory towards the event horizon. The null geodesic could then leave the apparent horizon and begin expanding, in violation of the focussing theorem.

Furthermore, in [16], Lowe constructs a related counterexample to the covariant entropy bound in the presence of a critically illuminated black hole. Critical illumination is the process in which matter is thrown into a black hole at exactly the same rate as energy is Hawking radiated away. In this scenario, the apparent horizon follows a null trajectory. If we pick the apparent horizon to be the generating surface for a lightsheet, then the lightsheet will coincide with the apparent horizon as long as we continue to critically illuminate the black hole. By critically illuminating the black hole sufficiently long, we can pass an arbitrary amount of matter through the lightsheet. In this way, the entropy of the matter passing through the lightsheet can be made larger than the area of the apparent horizon, thus violating the entropy bound.

Hence, the original Bousso bound only has a chance of holding in the classical regime. Once we include one-loop quantum effects, such as Hawking radiation, the bound fails. In the remaining sections of this chapter, we propose a modification of the Bousso bound which may hold in the semiclassical regime.

2.4 A quantum Bousso bound

The generalized Bousso bound, when specialized to black hole horizons, is equivalent to a classical limit of the generalized second law of thermodynamics (GSL). To see this, note that the portion of the event horizon lying between any two times consitutes a lightsheet. Since all matter falling into the black hole between those two times must pass through this lightsheet, the generalized entropy bound gives us the same information as the GSL. In particular, we learn that

$$\frac{1}{4}\Delta A_{\rm EH} \geq \Delta S_{\rm m}\,, \tag{2.4.1}$$

where $\Delta A_{\rm EH}$ is the change in event horizon area, and $\Delta S_{\rm m}$ is the entropy of the matter that fell in.

When quantum effects are included, the form (2.4.1) of the generalized second law is no longer valid. The quantum GSL states, roughly speaking, that the total entropy outside the black hole plus one quarter the area of the horizon (either event or apparent depending on the formulation) is non-decreasing. The entropy outside the black hole receives an important contribution from Hawking radiation. Therefore, we must augment the left hand side by the entropy of the Hawking radiation:

$$\frac{1}{4}\Delta A_{\mathrm{H}} + \Delta S_{\mathrm{hr}} \geq \Delta S_{\mathrm{m}} \tag{2.4.2}$$

In general, we do not know how to formulate, let alone prove, an exact form of the GSL in a full quantum theory of gravity. However, approximations to it have been formulated and demonstrated in a wide variety of circumstances [21]. The ΔS_{hr} term is crucial in these demonstrations, without which counterexamples may be easily found.

Since the GSL requires an additional term at the quantum level, and the GSL is a special case of the generalized Buosso bound, we should certainly expect that the Bousso bound will receive related quantum corrections. These corrections should reduce to ΔS_{hr} when the lightsheets are taken to be portions of event horizons. The problem is to precisely formulate the nature of these corrections.

In this context it is useful to think of the entropy in Hawking radiation as entanglement entropy. Evolution of the quantum fields on a fixed black hole geometry is a manifestly unitary process prior to singularity formation. Nevertheless entropy is created outside the black hole because the outgoing Hawking quanta are correlated with those that fall behind the horizon. When a region of space U is unobservable, we should trace the quantum state ψ over the modes in the unobservable region to obtain the observable density matrix ρ,

$$\rho = \mathrm{tr}_U \, |\psi\rangle\langle\psi| \, . \tag{2.4.3}$$

Since the full state is in principle not available to the observer, there is a de facto loss of information that can be characterized by the entanglement entropy

$$S_{\rm ent} = -{\rm tr}\,\rho \log \rho\,. \tag{2.4.4}$$

In general, this expression has divergences and requires further definition, which will be given below for the case of two dimensions.[1] Choosing U to be the region behind the horizon, we can therefore formally identify

$$\Delta S_{\rm hr} = \Delta S_{\rm ent}. \tag{2.4.5}$$

This motivates a natural guess for quantum corrections to the Bousso bound when the initial and final surfaces are closed. One should add to the area the entanglement entropy across the surface. Applying this modification to the classical Bousso bound (2.2.5) results in a quantum Bousso bound of the form:

$$\int_{L(B-B')} s \;\leq\; \frac{1}{4}A(B) + S_{\rm ent}(B) - \frac{1}{4}A(B') - S_{\rm ent}(B')\,. \tag{2.4.6}$$

Since we can not presently hope to solve this problem or even define this quantum bound in exact quantum gravity, in order to go further we need to identify a small expansion parameter for approximating the exact theory. A useful parameter, which systematically captures the quantum corrections of Hawking radiation, is provided by $\frac{1}{N}$, where N is the number of matter fields and $G_N N$ is held fixed [19]. In [23] it was shown in the two-dimensional RST model of black hole evaporation that the (suitably defined) GSL, incorporating the Hawking radiation as in (2.4.2), is valid. One might hope that a similar incorporation can save the Bousso bound.

In the process of the investigations in [23] it emerged that the sum $A + 4S_{\rm ent} \equiv A_{\rm qu}$ arises naturally in the theory as a kind of quantum-corrected area. In this chapter,

[1] UV divergences in this expression are absorbed by the renormalization of Newton's constant [22].

we propose that the required leading $\frac{1}{N}$ semiclassical correction to the generalized Bousso bound simply involves the replacement of the classical area with this quantum corrected area. A precise version of this statement will be formulated and proved in the RST model in section 2.6.

2.5 Proving the classical Bousso bounds

In this section we reproduce proofs of classical Bousso bounds. We first give a proof due to Bousso, Flanagan, and Marolf of the generalized Bousso bound in four dimensions [12].[2] This simplified proof follows from conditions on the initial entropy flux and an adiabaticity condition on the rate of change of the entropy flux which differ somewhat from the conditions assumed in [11]. We then describe a two dimensional version of the proof obtained by spherical reduction. A small modification of this gives a proof of the generalized Bousso bound in the classical CGHS model [19], which is then transcribed into Kruskal gauge for later convenience. The inclusion of quantum effects in the latter will be the subject of the next section.

2.5.1 Simplified proof in four dimensions

Following [11], the integral of the entropy flux s over the lightsheet can be written as

$$\int_{L(B-B')} s \;=\; \int_B d^2x\sqrt{h(x)}\int_0^1 d\lambda\, s(x,\lambda)\mathcal{A}(x,\lambda)\,. \qquad (2.5.1)$$

In this expression, we have chosen a coordinate system (x^1, x^2) on the spatial surface B, $h(x)$ is the determinant of the induced metric on B, and the affine parameter on each null geodesic of the lightsheet has been normalized so that $\lambda = 1$ is when the geodesic reaches B'. The function $\mathcal{A}(x,\lambda)$ is the area decrease factor for the geodesic

[2] We thank Raphael Bousso and Eanna Flanagan for explaining this proof prior to publication.

that begins at the point x on B. In terms of θ, it is given by

$$\mathcal{A} \equiv \exp\left[\int_0^\lambda d\tilde{\lambda}\, \theta(\tilde{\lambda})\right] . \tag{2.5.2}$$

The physical intuition for equation (2.5.1) is simple. As we parallel propagate a small coordinate patch of area $d^2x\sqrt{h(x)}$ from the point $(x, 0)$ on B to the point (x, λ) on the lightsheet, the area contracts to $d^2x\sqrt{h(x)}\mathcal{A}(x, \lambda)$. The proper three-dimensional volume of an infinitesimal cube of the lightsheet is $d^2x\, d\lambda\sqrt{h(x)}\mathcal{A}(x, \lambda)$, and this volume times $s(x, \lambda)$ gives the entropy flux passing through that cube. In order to prove the generalized entropy bound, it is sufficient to prove that

$$\int_0^1 d\lambda\, s(\lambda)\, \mathcal{A}(\lambda) \leq \frac{1}{4}(1 - \mathcal{A}(1)) \tag{2.5.3}$$

for each of the geodesics that comprise the lightsheet.

Using a mostly positive metric signature, the assumed entropy conditions are

i. $s' \leq 2\pi\, T_{ab}k^a k^b$

ii. $s(0) \leq -\frac{1}{4}\, \mathcal{A}'(0)$,

where we use the notation, both here and henceforth, that primes denote differentiation with respect to the affine parameter λ. Condition **i** is very similar to one of the conditions in [11]. It can be interpreted as the requirement that the rate of change of the entropy flux is less than the energy flux, which is a necessary condition for the thermodynamic approximation to hold. Condition **ii** requires only that the covariant entropy bound is not violated infinitesimally at the beginning of the lightsheet. Since the square root of $\mathcal{A}$ routinely appears in calculations, we borrow the notation of FMW and define

$$G \equiv \sqrt{\mathcal{A}} . \tag{2.5.4}$$

From the Raychaudhuri equation, we have that

$$T_{ab}k^a k^b = -\frac{1}{4\pi}\frac{G''(\lambda)}{G(\lambda)} - \frac{1}{8\pi}\sigma_{ab}\sigma^{ab} \leq -\frac{1}{4\pi}\frac{G''(\lambda)}{G(\lambda)}\,, \tag{2.5.5}$$

where σ_{ab} is the shear tensor, and the inequality follows from the fact that $\sigma_{ab}\sigma^{ab} \geq 0$ always. Now we see that

$$\begin{aligned}
s(\lambda) \quad &= \quad \int_0^\lambda d\tilde{\lambda}\, s'(\tilde{\lambda}) + s(0) \\
(i) \quad &\leq \quad 2\pi\int_0^\lambda d\tilde{\lambda} T_{ab}k^a k^b + s(0) \\
(eom) \quad &\leq \quad 2\pi\int_0^\lambda d\tilde{\lambda}\left(-\frac{1}{4\pi}\frac{G''(\tilde{\lambda})}{G(\tilde{\lambda})}\right) + s(0) \\
&= \quad \frac{1}{2}\left(\frac{G'(0)}{G(0)} - \frac{G'(\lambda)}{G(\lambda)}\right) - \frac{1}{2}\int_0^\lambda d\tilde{\lambda}\frac{G'(\tilde{\lambda})^2}{G(\tilde{\lambda})^2} + s(0) \\
(ii) \quad &\leq \quad -\frac{1}{2}\frac{G'(\lambda)}{G(\lambda)} - \frac{1}{2}\int_0^\lambda d\tilde{\lambda}\frac{G'(\tilde{\lambda})^2}{G(\tilde{\lambda})^2} \\
&\leq \quad -\frac{1}{2}\frac{G'(\lambda)}{G(\lambda)}\,.
\end{aligned}$$

Consequently,

$$\int_0^1 d\lambda\, s(\lambda)G(\lambda)^2 \leq -\frac{1}{2}\int_0^1 d\lambda\, G(\lambda)G'(\lambda) = \frac{1}{4}\left(\mathcal{A}(0) - \mathcal{A}(1)\right)\,. \tag{2.5.6}$$

We have shown that, given our entropy conditions, the entropy passing through a lightsheet is bounded by one quarter the difference in area of the two bounding spatial surfaces. This is precisely the statement of the generalized Bousso bound.

It is interesting to note that nowhere in the proof did we need to use the contracting lightsheet condition. The only indication that we should choose a contracting lightsheet comes from the boundary condition **ii**. We see from condition **ii** that, in order to allow a positive, future-directed entropy flux, the derivative of $\mathcal{A}$ must be negative. If the lightsheet were expanding at $\lambda = 0$, then a timelike entropy flux would have to be past-directed at $\lambda = 0$.

Note also that Bousso's entropy bound can be saturated only if $G' = 0$ for all λ. In light of the Raychaudhuri equation (2.5.5), we see that T_{ab} and the shear σ_{ab} must be zero everywhere along the lightsheet in order for G' to remain zero. The bound can be saturated only in this most trivial scenario. This will not be the case for other gravitational theories we will study, such as the CGHS dilaton model, where saturation of the bound can occur in the presence of matter.

2.5.2 Spherical reduction

Our goal is to study the entropy bound in two dimensional models where our semi-classical analysis will be greatly simplified. As a guide to what phenomenological conditions we should be using in 2D models, we will first rederive the previous proof for the purely spherical sector of 4D Einstein-Hilbert gravity.

We begin with the 4D Einstein-Hilbert action coupled to some matter Lagrangian density, $\mathcal{L}_m$:

$$\int d^4x \sqrt{-g^{(4)}} \left(\frac{R^{(4)}}{16\pi} + \mathcal{L}_m^{(4)} \right) . \tag{2.5.7}$$

Writing the four-dimensional metric as

$$(ds^2)^{(4)} = g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2\phi(x)} (d\vartheta^2 + \sin^2\vartheta \, d\varphi^2) \quad \mu, \nu \in \{0, 1\} , \tag{2.5.8}$$

and integrating over the angular coordinates we find the action is reduced to

$$\int d^2x \sqrt{-g} \left[e^{-2\phi} \left(\frac{1}{4} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} e^{2\phi} \right) + \mathcal{L}_m \right] . \tag{2.5.9}$$

Here, the 2D matter Lagrangian density $\mathcal{L}_m$ is related to $\mathcal{L}_m^{(4)}$ by

$$\mathcal{L}_m = 4\pi e^{-2\phi} \mathcal{L}_m^{(4)} . \tag{2.5.10}$$

From the equations of motion, we conclude that

$$k^a k^b T_{ab} = -e^{-\phi} k^a k^b \nabla_a \nabla_b e^{-\phi}\,, \tag{2.5.11}$$

whenever k is a null vector. In this expression, T is the energy-momentum tensor for $\mathcal{L}_m$, not $\mathcal{L}_m^{(4)}$.

It is clear from the four-dimensional metric that the classical "area" of a point in the 2D model is $A_{\rm cl} = 4\pi e^{-2\phi}$. However, had we only been given the action, we could identify the "area" of a point as being proportional to the factor multiplying the Ricci scalar in the Lagrange density. If that were not convincing enough, we could study the thermodynamics of a black hole solution of the two-dimensional model. In particular, we would first determine the mass of a stationary black hole solution and then compute the temperature of the Hawking radiation on this geometry (neglecting backreaction). Integrating the thermodynamic identity $dS = dM/T$ and identifying S as one-quarter the area of the event horizon, we arrive at an expression for the area of the event horizon in terms of the local values of the various fields there. We then designate this function of local fields as the expression that gives us the "area" of any point in the two-dimensional space.

Deriving the two-dimensional entropy conditions is a simple matter of rewriting the four-dimensional conditions in terms of two-dimensional tensors. For example, we replace $T_{ab}^{(4)}$ with $\frac{1}{4\pi}e^{2\phi}T_{ab}$. We are also interested in the two-dimensional entropy flux s_a which is related to the four-dimensional entropy flux $s_a^{(4)}$ by $s_a^{(4)} = \frac{1}{4\pi}e^{2\phi}s_a$. This relation is a simple consequence of the fact that the 2D flux at a point equals the 4D flux up to an overall factor of the area of the corresponding S^2. Replacing 4D tensors with 2D tensors, we arrive at the following entropy conditions:

i. $e^{-2\phi}(s\, e^{2\phi})' \leq 2\pi T_{ab}\, k^a k^b$

ii. $s(0) \leq -\frac{1}{4}A'_{\rm cl}(0)$

Note that we continue to use $s \equiv -k^a s_a$ and primes denoting $d/d\lambda$. Putting it all together, the derivation of the entropy bound goes through in the same way as it did in the 4D case. In detail, we find

$$\begin{aligned}
s(\lambda) &= e^{-2\phi(\lambda)} \int_0^\lambda d\tilde{\lambda} \left(s(\tilde{\lambda}) e^{2\phi(\tilde{\lambda})} \right)' + e^{-2\phi(\lambda)} s(0) e^{2\phi(0)} \\
(i) \qquad &\leq e^{-2\phi(\lambda)} \int_0^\lambda d\tilde{\lambda}\, 2\pi\, e^{2\phi(\tilde{\lambda})} k^a k^b T_{ab}(\tilde{\lambda}) + e^{-2\phi(\lambda)} s(0) e^{2\phi(0)} \\
(eom) \qquad &= -2\pi e^{-2\phi(\lambda)} \int_0^\lambda d\tilde{\lambda} \left(e^{-\phi(\tilde{\lambda})} \right)'' e^{\phi(\tilde{\lambda})} + e^{-2\phi(\lambda)} s(0) e^{2\phi(0)} \\
&= -2\pi e^{-2\phi(\lambda)} \left(-\phi'(\lambda) + \phi'(0) \right) - 2\pi e^{-2\phi(\lambda)} \int_0^\lambda d\tilde{\lambda} \left(\phi'(\tilde{\lambda}) \right)^2 + e^{-2\phi(\lambda)} s(0) e^{2\phi(0)} \\
(ii) \qquad &\leq -\pi \left(e^{-2\phi(\lambda)} \right)' - 2\pi e^{-2\phi(\lambda)} \int_0^\lambda d\tilde{\lambda} \left(\phi'(\tilde{\lambda}) \right)^2 \\
&\leq -\pi \left(e^{-2\phi(\lambda)} \right)'
\end{aligned}$$

Therefore,

$$\int_0^1 d\lambda\, s(\lambda) \leq \pi \left(e^{-2\phi(0)} - e^{-2\phi(1)} \right) = \frac{1}{4} \left(A_{\rm cl}(0) - A_{\rm cl}(1) \right) , \qquad (2.5.12)$$

which is exactly the 4D entropy bound, only derived from the 2D perspective.

2.5.3 CGHS

Although we have derived an entropy bound in a 2D model using 2D entropy conditions, we were guaranteed success since we had spherically reduced a successful 4D proof. We now attempt to apply the same entropy conditions to another 2D dilaton gravity model, namely the CGHS model [19]. The CGHS model can also be derived as the spherical reduction of a 4D model, but with charges. In what follows, we will work purely at the 2D level without any recourse to higher-dimensional physics. The

CGHS action coupled to N conformal matter fields with Lagrangian density $\mathcal{L}_m$ is

$$\int d^2x\sqrt{-g}\left[e^{-2\phi}(R+4(\nabla\phi)^2+4)+\mathcal{L}_m\right] . \tag{2.5.13}$$

For a null vector k^a, the equations of motion give

$$k^ak^bT_{ab}=-2e^{-\phi}k^ak^b\nabla_a\nabla_b e^{-\phi}+2k^ak^b\nabla_a e^{-\phi}\nabla_b e^{-\phi} . \tag{2.5.14}$$

To determine the classical "area" of a point, we look at the coefficient of the Ricci scalar and learn that it is proportional to $e^{-2\phi}$. By studying black hole thermodynamics, the constant of proportionality can be fixed as $A_{\rm cl}=8e^{-2\phi}$.

To prove the entropy bound, we start with the following assumptions:

i. $e^{-2\phi}(s\,e^{2\phi})'\leq 2\,T_{ab}\,k^ak^b$

ii. $s(0)\leq -\frac{1}{4}A_{\rm cl}'(0)$

We will continue to use $s\equiv -k^as_a$ and primes denoting $d/d\lambda$. Putting it all together, we find

$$\begin{array}{rcl}
s(\lambda) & = & e^{-2\phi(\lambda)}\displaystyle\int_0^\lambda d\tilde\lambda\left(s(\tilde\lambda)\,e^{2\phi(\tilde\lambda)}\right)'+e^{-2\phi(\lambda)}s(0)e^{2\phi(0)}\\
(i) & \leq & 2\,e^{-2\phi(\lambda)}\displaystyle\int_0^\lambda d\tilde\lambda\,e^{2\phi(\tilde\lambda)}k^ak^bT_{ab}(\tilde\lambda)+e^{-2\phi(\lambda)}s(0)e^{2\phi(0)}\\
(eom) & = & -4\,e^{-2\phi(\lambda)}\displaystyle\int_0^\lambda d\tilde\lambda\left(e^{-\phi(\tilde\lambda)}\right)''e^{\phi(\tilde\lambda)}+4\,e^{-2\phi(\lambda)}\displaystyle\int_0^\lambda d\tilde\lambda\left(\phi'(\tilde\lambda)\right)^2+e^{-2\phi(\lambda)}s(0)e^{2\phi(0)}\\
 & = & -4\,e^{-2\phi(\lambda)}\left(-\phi'(\lambda)+\phi'(0)\right)+e^{-2\phi(\lambda)}s(0)e^{2\phi(0)}\\
(ii) & \leq & -2\left(e^{-2\phi(\lambda)}\right)' .
\end{array}$$

Therefore, we find the desired relation:

$$\int_0^1 d\lambda\,s(\lambda)\leq 2\left(e^{-2\phi(0)}-e^{-2\phi(1)}\right)=\frac{1}{4}\left(A_{\rm cl}(0)-A_{\rm cl}(1)\right) . \tag{2.5.15}$$

2.5.4 CGHS in Kruskal gauge

In the previous section, we derived the CGHS entropy bound with manifestly covariant equations of motion and entropy conditions. However, once we add the one-loop trace anomaly, we are only able to obtain local equations of motion in conformal gauge. Furthermore, our calculations are greatly simplified in Kruskal gauge. Therefore, it behooves us to rederive the CGHS result in Kruskal gauge.

We will assume that the lightsheet moves in the decreasing x^+ direction. Our results for this past-directed x^+ lightsheet generalize simply to the other three possible lightsheet directions. Working with the x^+ lightsheet, we will be interested in the following equation of motion:

$$T_{++} = -2e^{-\phi}\nabla_+\nabla_+ e^{-\phi} + 2\nabla_+ e^{-\phi}\nabla_+ e^{-\phi}\,. \tag{2.5.16}$$

In conformal gauge, the RHS can be written as $2e^{-2\phi}\left(\partial_+\partial_+\phi - 2\partial_+\rho\partial_+\phi\right)$. In Kruskal gauge, we set $\rho = \phi$, so this becomes

$$T_{++} = -\partial_+\partial_+ e^{-2\phi}\,. \tag{2.5.17}$$

Since $k^+ = -\partial x^+/\partial\lambda = e^{-2\phi}$ in Kruskal gauge, our entropy conditions can be rewritten in Kruskal gauge coordinates as

i. $\partial_+ s_+ \leq 2\,T_{++}$

ii. $-s_+(x_0^+) \leq \frac{1}{4}\,\partial_+ A_{\text{cl}}(x_0^+)$.

Recall that $s \equiv -k^+ s_+$, so $-s_+$ is positive so long as the proper entropy flux s is positive.

Applying these conditions, we find

$$\begin{aligned}
-s_+(x^+) &= \int_{x^+}^{x_0^+} d\tilde{x}^+ \partial_+ s_+(\tilde{x}^+) - s_+(x_0^+) \\
(i) \quad &\leq 2\int_{x^+}^{x_0^+} d\tilde{x}^+ T_{++}(\tilde{x}^+) - s_+(x_0^+) \\
(eom) \quad &= 2\,\partial_+ e^{-2\phi}\big]_{x_0^+}^{x^+} - s_+(x_0^+) \\
(ii) \quad &\leq 2\,\partial_+ e^{-2\phi(x^+)} \,.
\end{aligned}$$

We find that

$$\int_0^1 d\lambda\, s(\lambda) = \int_{x_0^+}^{x_1^+} d\tilde{x}^+ s_+(\tilde{x}^+) = \int_{x_1^+}^{x_0^+} d\tilde{x}^+ \left(-s_+(\tilde{x}^+)\right) \leq \frac{1}{4}\left(A_{\rm cl}(0) - A_{\rm cl}(1)\right) \,. \tag{2.5.18}$$

Had we chosen the future-directed x^+ lightsheet, then we would have $k^+ = \partial x^+/\partial\lambda = e^{-2\phi}$ and our entropy conditions would have been $\partial_+\left(-s_+\right) \leq 2\,T_{++}$ and $-s_+(x_0^+) \leq -\frac{1}{4}\,\partial_+ A_{\rm cl}(x_0^+)$. The extension to x^- lightsheets is trivial.

2.6 Stating and proving a quantum Bousso bound

The classical CGHS action is

$$S_{\rm CGHS} = \int d^2x\sqrt{-g}\left[e^{-2\phi}(R + 4(\nabla\phi)^2 + 4) + \mathcal{L}_m\right] \,. \tag{2.6.1}$$

For N conformal matter fields, Hawking radiation and its backreaction on the geometry can be accounted for by adding to the classical CGHS action the nonlocal term

$$S_{\rm PL} = -\frac{N}{48}\int d^2x\sqrt{-g(x)}\int d^2x'\sqrt{-g(x')}R(x)G(x,x')R(x') \,, \tag{2.6.2}$$

where $G(x,x')$ is the Green's function for ∇^2. This is a one loop quantum correction but it nevertheless contributes at leading order in a $\frac{1}{N}$ expansion. At the one loop level, there is the freedom of also adding a local counterterm to the action. The large N theory becomes analytically soluble if we add the following judiciously chosen local counterterm to the action [17, 18]:

$$S_{\mathrm{ct}} = -\frac{N}{24}\int d^2x\sqrt{-g}\phi R\,. \tag{2.6.3}$$

The full action for the RST model is then

$$S_{\mathrm{RST}} = S_{\mathrm{CGHS}} + S_{\mathrm{PL}} + S_{\mathrm{ct}}\,. \tag{2.6.4}$$

We can once again choose Kruskal gauge, but this time $\rho = \phi + \frac{1}{2}\log(N/12)$. In conformal and Kruskal gauges, the equations of motion become

$$\partial_+\partial_-\Omega = -1\,, \tag{2.6.5}$$

and

$$\partial_\pm^2\Omega = -\frac{12}{N}T_{\pm\pm} - t_\pm\,, \tag{2.6.6}$$

where

$$\Omega = \frac{12}{N}e^{-2\phi} + \frac{\phi}{2} + \frac{1}{4}\log\frac{N}{48}\,. \tag{2.6.7}$$

The $t_\pm$ term in (2.6.6) accounts for the normal-ordering ambiguity. We wish to consider semiclassical excitations built on the vacuum state which has no positive frequency excitations according to inertial observers on $\mathcal{I}^-$. These inertial coordinates, $\sigma^\pm$, are related to the Kruskal coordinates by

$$x^+ = e^{\sigma^+}\,, \quad x^- = -e^{-\sigma^-}\,. \tag{2.6.8}$$

For coherent states built on this σ vacuum, $t_\pm = 0$ in σ coordinates. Its value in

Kruskal coordinates is given by the Schwarzian transformation law as

$$t_\pm = -\frac{1}{4(x^\pm)^2}\,. \tag{2.6.9}$$

As worked out in [23], the classical "area" of a point in the RST model is

$$A_{\rm cl} = 8e^{-2\phi} - \frac{N}{3}\phi - \frac{N}{6} - \frac{N}{6}\log\frac{N}{48}\,. \tag{2.6.10}$$

For coherent states built on the σ vacuum, the renormalized entanglement entropy across a point has a local contribution

$$S_{\rm ent} = \frac{N}{6}\left(\phi + \frac{1}{2}\log\frac{N}{12} + \frac{1}{2}\log\left(-x^+x^-\right)\right)\,. \tag{2.6.11}$$

The full entanglement entropy also has an infrared term which is not locally associated to the horizon and so is not included here. See [23] for a detailed derivation and discussion of these points.

Now, Ω can be written as

$$\Omega = \frac{3}{2N}\left(A_{\rm cl} + 4S_{\rm ent}\right) - \frac{1}{2}\log(-x^+x^-) - \log 2 + \frac{1}{4}\,. \tag{2.6.12}$$

Differentiating, we obtain

$$\partial_+\Omega + \frac{1}{2x^+} = \frac{3}{2N}\partial_+A_{\rm qu}\,, \tag{2.6.13}$$

where $A_{\rm qu} = A_{\rm cl} + 4S_{\rm ent}$.

When analyzing the RST model, we will leave entropy condition **i** unchanged. In the formulation of **ii**, we will replace $A_{\rm cl}$ with $A_{\rm qu} \equiv A_{\rm cl} + 4S_{\rm ent}$. In Kruskal coordinates, these become

i. $\partial_+ s_+ \leq 2\,T_{++}$

ii. $-s_+(x_0^+) \leq \frac{1}{4}\partial_+ A_{\text{qu}}(x_0^+)$

Application of these conditions results in

$$\begin{aligned}
-s_+(x^+) &= \int_{x^+}^{x_0^+} d\tilde{x}^+ \partial_+ s_+(\tilde{x}^+) - s_+(x_0^+) \\
(i) \quad &\leq 2\int_{x^+}^{x_0^+} d\tilde{x}^+ T_{++}(\tilde{x}^+) - s_+(x_0^+) \\
(eom) \quad &= \frac{N}{6}\left(\partial_+\Omega + \frac{1}{4x^+}\right)\Big]_{x_0^+}^{x^+} - s_+(x_0^+) \\
&= \left(\frac{1}{4}\partial_+ A_{\text{qu}} - \frac{N}{24x^+}\right)\Big]_{x_0^+}^{x^+} - s_+(x_0^+) \\
(ii) \quad &\leq \frac{1}{4}\partial_+ A_{\text{qu}}(x^+) - \frac{N}{24}\frac{1}{x^+} + \frac{N}{24}\frac{1}{x_0^+} \\
&\leq \frac{1}{4}\partial_+ A_{\text{qu}}(x^+)\,.
\end{aligned}$$

We find

$$\int_0^1 d\lambda\, s(\lambda) = \int_{x_0^+}^{x_1^+} d\tilde{x}^+ s_+(\tilde{x}^+) = \int_{x_1^+}^{x_0^+} d\tilde{x}^+ \left(-s_+(\tilde{x}^+)\right) \leq \frac{1}{4}\left(A_{\text{qu}}(0) - A_{\text{qu}}(1)\right)\,. \tag{2.6.14}$$

With our entropy conditions, we see that the covariant entropy bound is satisfied once we replace A_{cl} with A_{qu}.

It is interesting to note that the quantum Bousso bound can not be saturated for coherent states built on the σ vacuum. The obstruction to saturation is the term $\frac{N}{24}\left(\frac{1}{x_0^+} - \frac{1}{x^+}\right)$ that shows up in the calculation of $-s_+(x^+)$. However, had we built our state on top of the Kruskal vacuum (i.e., the Hartle-Hawking state), we would have $t_+ = 0$ and $S_{\text{ent}} = \frac{N}{6}(\phi + \frac{1}{2}\log\frac{N}{12})$. As a result, both our equations of motion and our definition of A_{qu} would change in a way that eliminates the $\frac{N}{24}\left(\frac{1}{x_0^+} - \frac{1}{x^+}\right)$ term from the calculations. The quantum Bousso bound will then be saturated any time the two entropy conditions are saturated.

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Chapter 3

Descent Relations in Type 0A/0B

3.1 Introduction

In this chapter, we gain further insight into type 0 D-branes by working out the descent relations for type 0 theories. Sen's descent relations in the type II theories relate different D-branes through operations of orbifolding and tachyon kinking. These relations form an interlocking chain of relationships between the different types of D-branes. Although the type 0 theories are in many ways similar to the type II theories, it is not immediately clear how one should draw the descent relation diagram since type 0 theories have twice the number of D-branes. This problem is addressed in sections 3.4 through 3.6.

Sections 3.2 and 3.3 serve as very brief introductions to the type 0 theories and their D-brane content. In section 3.4, we review the descent relations in type II theories and we manage to rule out certain combinations of type 0 D-branes from having any starring role in the type 0 descent relations. In sections 3.5 and 3.6, we uncover how the type 0 D-branes are related via orbifolds and kinks, respectively. By the end of section 3.6, we have pieced together the type 0 descent relations.

Section 3.7 demonstrates the fundamental distinction between the two types of D-branes in type 0 theories. We show in this last section that the two types of D-branes, D+ branes and D− branes, have opposite charges with respect to all (NS−,NS−) fields. We will also show how a general disk amplitude with a D+ relates to the same amplitude with a D−.

3.2 Perturbative Spectrum

Type II superstring theories are composed of left and right moving pieces which reside in one of four sectors, NS± and R±. The + and − here denote the value of the worldsheet fermion number operator, $(-1)^F$, not to be confused with the $(-1)^{F_L^s}$ operator to be introduced later. At first blush, it appears as though there are on the order of 2^{16} possible string theories, each factor of 2 coming from whether or not a given theory contains a particular combination of sectors. Several consistency conditions pare this enormous number of possibilities to only four. Two of these are the type IIA and IIB theories. The other two are the less familiar type 0A and 0B theories.

The consistency conditions are as follows (for a review, see [24]):

Level matching: The first condition we use to rule out some theories is the level matching condition $L_0 = \tilde{L}_0$. The NS− sector has half-integer levels while the NS+, R+, and R− have integer levels. Therefore, NS− can not be paired with any of the other three sectors.

Mutual locality: All pairs of vertex operators must be mutually local. That is, the phase obtained by taking one vertex operator in a circle around the other must be unity or else there is phase ambiguity in the amplitude.

Closed OPE: The OPE of the vertex operators in the theory must be in terms of vertex operators that are also present in the theory.

Modular invariance: Modular invariance requires that there be at least one left moving R sector and at least one right moving R sector.

The only four theories that satisfy these simple consistency requirements are the type IIA and IIB theories,

$$\begin{array}{lllll} \text{IIA:} & (\text{NS}+,\text{NS}+) & (\text{R}+,\text{R}-) & (\text{NS}+,\text{R}-) & (\text{R}+,\text{NS}+) \\ \text{IIB:} & (\text{NS}+,\text{NS}+) & (\text{R}+,\text{R}+) & (\text{NS}+,\text{R}+) & (\text{R}+,\text{NS}+) \end{array} \tag{3.2.1}$$

and the type 0A and 0B theories,

$$\begin{array}{lllll} \text{0A:} & (\text{NS}+,\text{NS}+) & (\text{NS}-,\text{NS}-) & (\text{R}+,\text{R}-) & (\text{R}-,\text{R}+) \\ \text{0B:} & (\text{NS}+,\text{NS}+) & (\text{NS}-,\text{NS}-) & (\text{R}+,\text{R}+) & (\text{R}-,\text{R}-)\,. \end{array} \tag{3.2.2}$$

The perturbative spectra of the type 0 theories contain no spacetime fermions. In the NS-NS sectors, the low-lying states are the tachyon from (NS−,NS−) and the graviton, antisymmetric tensor, and dilaton from (NS+,NS+). The type 0 theories have twice as many massless R-R states as the type II theories. In particular, type 0A has two R-R 1-forms and two R-R 3-forms; type 0B has two R-R scalars, two R-R 2-forms, and one R-R 4-form with an unconstrained 5-form field strength.

3.3 D-branes

The fact that the type 0 theories have twice as many R-R fields as the type II theories is an indication that there may be twice as many stable D-branes in type 0 as compared to type II. This turns out to be correct and can be understood quite directly by examining D-branes in the boundary state formalism (for a review, see [25]). In this formalism, D-branes are represented by boundary states for the physical closed strings. These boundary states are themselves coherent closed string states.

In both the type II and type 0 theories, there are four types of boundary states for each p,

$$|Bp,+\rangle_{\text{NS-NS}}\,, \quad |Bp,-\rangle_{\text{NS-NS}}\,, \quad |Bp,+\rangle_{\text{R-R}}\,, \quad |Bp,-\rangle_{\text{R-R}}\,. \tag{3.3.1}$$

The $+$ and $-$ denote the boundary conditions on the worldsheet fermions and superghosts as in equations (3.6.9). Linear combinations of these states must be taken to form D-brane boundary states which, in turn, must be GSO-invariant and must satisfy certain consistency conditions [25].

The D-brane boundary states in the type 0 theories are as follows:

$$\left.\begin{array}{rcl} |Dp,+\rangle &=& |Bp,+\rangle_{\text{NS-NS}} + |Bp,+\rangle_{\text{R-R}} \\ |Dp,-\rangle &=& |Bp,-\rangle_{\text{NS-NS}} + |Bp,-\rangle_{\text{R-R}} \\ |\overline{D}p,+\rangle &=& |Bp,+\rangle_{\text{NS-NS}} - |Bp,+\rangle_{\text{R-R}} \\ |\overline{D}p,-\rangle &=& |Bp,-\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{R-R}} \end{array}\right\} \text{for p even (odd) in 0A (0B)} \tag{3.3.2}$$

$$\left.\begin{array}{rcl} |\widehat{Dp},+\rangle &=& |Bp,+\rangle_{\text{NS-NS}} \\ |\widehat{Dp},-\rangle &=& |Bp,-\rangle_{\text{NS-NS}} \end{array}\right\} \text{for p odd (even) in 0A (0B).} \tag{3.3.3}$$

Using η to denote ± 1, the $|Dp,\eta\rangle$ states correspond to stable D-branes. We see from the minus sign in front of the R-R boundary states that the $|\overline{D}p,\eta\rangle$ states correspond to stable anti-D-branes. The $|\widehat{Dp},\eta\rangle$ states correspond to unstable D-branes.

Let us pause for a second to make a remark on D-brane stability. The condition for stability is that the spectrum of open strings on the D-brane does not contain a tachyon. It is important not to confuse this condition with being BPS. Of course, none of the D-branes can be BPS in the type 0 theories since there is no supersymmetry to begin with; there are no fermions in the absence of D-branes. It just so happened for D-branes in the type II theories that the conditions of stability and BPS coincided.

It will be important for our purposes to find the spectra of open strings living on or between D-branes. The details can be found in Appendix A and the results for type

0 D-branes are given in tables 1 and 2. The spectra in table 1 can be extrapolated to all possibilities by noting that a given spectrum is invariant under the replacements D $\leftrightarrow$ $\overline{\mathrm{D}}$ and/or $+ \leftrightarrow -$. For example, from the first line of table 1, we see that the open strings beginning on a Dp+ and ending on a Dp+ are NS+. Therefore, the strings beginning on a $\overline{\mathrm{D}}$p+ and ending on a $\overline{\mathrm{D}}$p+ are NS+. Similarly, strings beginning on a Dp− and ending on a Dp− (or beginning on a $\overline{\mathrm{D}}$p− and ending on a $\overline{\mathrm{D}}$p−) are also NS+.

Open Spectrum on Stable D-branes (p odd in 0B, p even in 0A)		
$\sigma = 0$	$\sigma = \pi$	Spectrum
Dp+	Dp+	NS+
Dp+	$\overline{\mathrm{D}}$p+	NS−
Dp+	Dp−	R+
Dp+	$\overline{\mathrm{D}}$p−	R−

Table 1: All other cases obtained by one or both of the following operations under which the spectrum is invariant: $+ \leftrightarrow -$, D $\leftrightarrow$ $\overline{\mathrm{D}}$.

We see that there are two tachyons among the open strings stretched between a $|Dp, \eta\rangle$ and a $|\overline{D}p, \eta\rangle$. One tachyon starts (at $\sigma = 0$) on the $|Dp, \eta\rangle$ and ends (at $\sigma = \pi$) on the $|\overline{D}p, \eta\rangle$, and the other tachyon starts on the $|\overline{D}p, \eta\rangle$ and ends on the $|Dp, \eta\rangle$. This indicates an instability in the D$\overline{\mathrm{D}}$ pair.

Open Spectrum on Unstable D-branes (all p in 0A and 0B)		
$\sigma = 0$	$\sigma = \pi$	Spectrum
$\widehat{\mathrm{Dp}}$+	$\widehat{\mathrm{Dp}}$+	NS+, NS−
$\widehat{\mathrm{Dp}}$+	$\widehat{\mathrm{Dp}}$−	R+, R−

Table 2: All other cases obtained by $+ \leftrightarrow -$ under which the spectrum is invariant.

We see in table 2, as expected, that there is a tachyon living on the unstable $|\widehat{Dp}, \eta\rangle$ D-branes.

3.4 Descent Relations

Sen's descent relations give relations between different D-brane configurations in the type II theories (for a review, see [14]). The two important operations are orbifolding by $(-1)^{F_L^s}$, where F_L^s is the spacetime fermion number of the left-movers, and kinking the tachyon field that lives on unstable configurations of D-branes. Starting with a coincident D(2p)$\overline{\text{D}}$(2p) pair in type IIA, orbifolding by $(-1)^{F_L^s}$ yields an unstable $\widehat{\text{D(2p)}}$ in type IIB. Orbifolding one more time leaves us with a stable D(2p) in the type IIA theory. Starting again with the D(2p)$\overline{\text{D}}$(2p) pair in type IIA, but this time kinking the tachyon field that lives on the D-branes, we are left with an unstable D$\widehat{\text{(2p−1)}}$ in type IIA. Kinking the remaining tachyon field gives us a stable D(2p−2) in type IIA. The results are similar if we start with a D(2p+1)$\overline{\text{D}}$(2p+1) pair in type IIB. In fact, the descent relations form an interlocking chain as shown in figure 1.

$$
\begin{array}{cccccccc}
 & & & & & \downarrow & & \downarrow \\
 & & & \text{IIB D(2p+1)}\overline{\text{D}}\text{(2p+1)} & \rightarrow & \text{IIA D}\widehat{\text{(2p+1)}} & \rightarrow & \text{IIB D(2p+1)} \\
 & & & \downarrow & & \downarrow & & \\
 & \text{IIA D(2p)}\overline{\text{D}}\text{(2p)} & \rightarrow & \text{IIB }\widehat{\text{D(2p)}} & \rightarrow & \text{IIA D(2p)} & & \\
 & \downarrow & & \downarrow & & & & \\
\rightarrow & \text{IIA D}\widehat{\text{(2p−1)}} & \rightarrow & \text{IIB D(2p−1)} & & & & \\
 & \downarrow & & & & & & \\
\rightarrow & \text{IIA D(2p−2)} & & & & & &
\end{array}
$$

Figure 1: Descent relations for the type II theories. Horizontal arrows denote modding by $(-1)^{F_L^s}$. Vertical arrows denote the tachyonic kink.

The natural question at this point is what the analogue of the descent relations is for the type 0 theories. Starting with a D(2p)$\overline{\text{D}}$(2p) in type 0A, we have four possibilities to consider: a choice of + or − for each of the two branes. Then, once we orbifold (kink), we must figure out whether we get $\widehat{\text{D(2p)}}$+ or $\widehat{\text{D(2p)}}$− ($\text{D}\widehat{(2\text{p}-1)}$+ or $\text{D}\widehat{(2\text{p}-1)}$−). For a discussion of the differences between D+ and D− branes, see section 3.7.

In the type II descent relations, every time we orbifold or kink we effectively remove one of the tachyonic degrees of freedom. A complex tachyon lives on the D$\overline{\text{D}}$ pair; orbifolding or kinking once gives an unstable D-brane with a real tachyon; orbifolding or kinking one more time gives a stable D-brane with no tachyon field. With this observation, we can quickly rule out two of the choices for the D$\overline{\text{D}}$ pair in the type 0 case. Since the open string tachyon arises from the NS− sector, we see from table 1 that only the Dp+$\overline{\text{D}}$p+ and Dp−$\overline{\text{D}}$p− pairs for p odd in 0B (even in 0A) have tachyon fields living on them.

Holding out some hope for the Dp+$\overline{\text{D}}$p− pair, let us see if there is any room in the type 0 descent relations for this object. Clearly, we can not consider a tachyon kink since there is no tachyonic kink on this pair of D-branes: from table 1, we see that there are NS+ strings living on each of the D-branes and R− strings stretched between the two. Perhaps we can orbifold this pair of D-branes by $(-1)^{F_L^s}$. However, one can take the $(-1)^{F_L^s}$ orbifold in the presence of D-branes only if that configuration of D-branes is invariant under $(-1)^{F_L^s}$. For example, in the type II theories, $(-1)^{F_L^s}|D(2p)\rangle = |\overline{D}(2p)\rangle$ and $(-1)^{F_L^s}|\overline{D}(2p)\rangle = |D(2p)\rangle$, so we were able to orbifold the D$\overline{\text{D}}$ pairs. Since

$$\begin{aligned}
(-1)^{F_L^s}|Bp,\pm\rangle_{\text{NS-NS}} &= |Bp,\pm\rangle_{\text{NS-NS}}\,, \\
(-1)^{F_L^s}|Bp,\pm\rangle_{\text{R-R}} &= -|Bp,\pm\rangle_{\text{R-R}}\,,
\end{aligned} \qquad (3.4.1)$$

we see from (3.3.2) that in the type 0 theories

$$
\begin{aligned}
(-1)^{F_L^s}|Dp+\rangle &= |\overline{D}p+\rangle\,, \\
(-1)^{F_L^s}|Dp-\rangle &= |\overline{D}p-\rangle\,,
\end{aligned}
\tag{3.4.2}
$$

and

$$
\begin{aligned}
(-1)^{F_L^s}|\overline{D}p+\rangle &= |Dp+\rangle\,, \\
(-1)^{F_L^s}|\overline{D}p-\rangle &= |Dp-\rangle\,.
\end{aligned}
\tag{3.4.3}
$$

This means that the coincident Dp+$\overline{\text{D}}$p− pair is not invariant under $(-1)^{F_L^s}$ and we no longer consider it as a potential participant in the type 0 descent relations. Fortunately, the Dp+$\overline{\text{D}}$p+ and Dp−$\overline{\text{D}}$p− pairs *are* invariant under $(-1)^{F_L^s}$, so we will be able to interpret the orbifold as a projection of the open string states.

3.5 $(-1)^{F_L^s}$ Orbifold

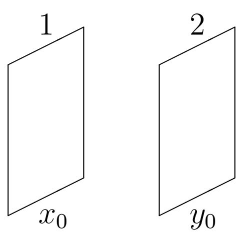
Here we will consider what happens to the coincident D(2p)+$\overline{\text{D}}$(2p)+ pair in type 0A under the $(-1)^{F_L^s}$ orbifold. First, let us look at the spacetime bulk far from the D-branes. Locally, this is just type 0A without any open strings. Taking the orbifold of type 0A by $(-1)^{F_L^s}$ gives the type 0B theory, and vice versa (see Appendix B for details).

As we have already noted in equations (3.4.2) and (3.4.3) , $(-1)^{F_L^s}$ switches the D(2p)+ and $\overline{\text{D}}$(2p)+, so its action on the Chan-Paton factors is

$$
\Lambda \to \sigma_1 \Lambda \sigma_1^{-1}\,. \tag{3.5.1}
$$

Of the four Chan-Paton factors, I, σ_1, σ_2, and σ_3, only I and σ_1 are invariant under this operation. Therefore, the open strings with CP factors I and σ_1 are kept and those with CP factors σ_2 and σ_3 are thrown out.

We can see that this new object, the result of orbifolding D(2p)+$\overline{\text{D}}$(2p)+, is a single brane since the degrees of freedom corresponding to the relative positions of the original D-branes have been projected out. The position coordinates corresponding to their respective CP factors are as given below.



$$\begin{aligned} 1-1 &\;:\; X = x_0 + \ldots \\ 2-2 &\;:\; X = y_0 + \ldots \\ 1-2 &\;:\; X = x_0 + \frac{\sigma}{\pi}(y_0 - x_0) + \ldots \\ 2-1 &\;:\; X = y_0 + \frac{\sigma}{\pi}(x_0 - y_0) + \ldots \end{aligned}$$

Writing out the lowest order degrees of freedom in terms of Chan-Paton factors, we find that we can regroup them as

$$\begin{aligned} & x_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + y_0 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + [x_0 + \frac{\sigma}{\pi}(y_0 - x_0)] \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + [y_0 + \frac{\sigma}{\pi}(x_0 - y_0)] \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ &= \frac{1}{2}(x_0 + y_0) I + \frac{1}{2}(x_0 - y_0)\sigma_3 + \frac{1}{2}(x_0 + y_0)\sigma_1 + \frac{1}{2}[-i(x_0 - y_0) + \frac{2i\sigma}{\pi}(x_0 - y_0)]\sigma_2 \, . \end{aligned} \tag{3.5.2}$$

The $(x_0 - y_0)$ degree of freedom multiplies only σ_2 and σ_3, which are projected out.

After orbifolding, we are left with a (2p)-brane in the type 0B theory with NS+ strings (corresponding to I) and NS$-$ strings (corresponding to σ_1) living on it. This identifies the object as either $\widehat{\text{D(2p)}}$+ or $\widehat{\text{D(2p)}}-$. In order to distinguish between these two options, we look at the coupling of this (2p)-brane to the (NS$-$,NS$-$) tachyon and compare it to the coupling of the $\widehat{\text{D(2p)}}$+ and $\widehat{\text{D(2p)}}-$ to the (NS$-$,NS$-$) tachyon. But first we must determine what these couplings are.

We know from [26] that *stable* D-branes in the type 0 theories have the term

$$-\frac{T_p q\overline{q}}{4}\int d^{p+1}\sigma\, T(X) \tag{3.5.3}$$

in their low energy effective action, where T is the closed string tachyon, and q and $\overline{q}$ are the D-brane's charges under the massless R-R fields C and $\overline{C}$. The R-R charges of stable D-branes in the type 0 theories are given in table 3. Notice that $q\overline{q} = \eta$.

Stable Dp R-R Charges (p odd in 0B, p even in 0A)		
	q	$\overline{q}$
Dp+	1	1
$\overline{\text{D}}$p+	−1	−1
Dp−	1	−1
$\overline{\text{D}}$p−	−1	1

Table 3

We know from cylinder diagrams between D-branes that the *unstable* $\widehat{\text{D+}}$ and $\widehat{\text{D}-}$ have opposite tachyon charge [27], but this can not tell us how to assign the charges to the two types of D-branes. The solution to this can be found by comparing tachyon tadpole calculations for the stable and unstable D-branes.

The amplitude [28, 29] for a stable Dp+ to emit a tachyon is

$$\begin{aligned}
\langle T,k|Dp,+\rangle &= \langle T,k|(|Bp,+\rangle_{\text{NS-NS}} + |Bp,+\rangle_{\text{R-R}}) \\
&= \langle T,k|Bp,+\rangle_{\text{NS-NS}} \\
&= \langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|Bp,+\rangle_{\text{NS-NS}} \\
&= \frac{T_p}{2}\langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|B_X\rangle|B_{\text{gh}}\rangle|B_\psi,\eta\rangle_{\text{NS-NS}}|B_{\text{sgh}},\eta\rangle_{\text{NS-NS}}\,,
\end{aligned} \tag{3.5.4}$$

where k is perpendicular to the D-brane. Now consider an unstable $\widehat{\text{D(p}-1)}$+ that is extended in p−1 of the same directions as the Dp+. The amplitude for an unstable $\widehat{\text{D(p}-1)}$+ to emit a tachyon in the same direction is

$$\begin{aligned} \langle T, k|\widehat{D(p-1)}, +\rangle &= \langle T, k|B(p-1), +\rangle_{\text{NS-NS}} \\ &= \frac{T_{p-1}}{2}\langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|B_X\rangle'|B_{\text{gh}}\rangle|B_\psi, \eta\rangle'_{\text{NS-NS}}|B_{\text{sgh}}, \eta\rangle_{\text{NS-NS}}. \end{aligned} \tag{3.5.5}$$

The only difference between (3.5.4) and (3.5.5) is the normalization and the matter part of the boundary state. Both T_p and T_{p-1} are positive constants. The difference between $|B_X\rangle'$ and $|B_X\rangle$ is a minus sign on one of the X fields which does not get contracted with the $e^{ik\cdot X}$ of the tachyon since k is perpendicular to the Dp+. The difference between $|B_\psi\rangle'$ and $|B_\psi\rangle$ is a minus sign on one of the ψ fields, but none of the ψ fields in the boundary state get contracted with anything in the tachyon vertex operator. Therefore, the tachyon charge of the unstable $\widehat{\text{D(p}-1)}$+ is related to the charge of the stable Dp+ by a factor of T_{p-1}/T_p, so the tachyon tadpole term in an unstable $|\widehat{D(p-1)}, \eta\rangle$ brane's low energy effective action is

$$-\frac{T_{p-1}\eta}{4}\int d^{p+1}\sigma\, T(X)\,. \tag{3.5.6}$$

Note, by comparing (3.5.3) and (3.5.6), that the Dp+ and the $\widehat{\text{D(p}-1)}$+ couple with the same sign to the closed string tachyon.

Since both the closed string tachyon and the NS-NS boundary state part of the D-branes both reside in the (NS,NS) sector which is unaffected by the orbifold, the coupling of the brane to the tachyon should be unchanged. This means that the D(2p)+$\overline{\text{D}}$(2p)+ in type 0A gets orbifolded to the $\widehat{\text{D(2p)}}$+ of the type 0B.

We can understand the orbifold at the level of boundary states by considering the emission and reabsorption of closed strings by the D(2p)+$\overline{\text{D}}$(2p)+ pair. To simplify our equations, we introduce the shorthand notation

$$\langle\langle\Lambda\rangle\rangle \equiv \int dl \begin{pmatrix} |D(2p){+}\rangle \\ |\overline{D}(2p){+}\rangle \end{pmatrix}^{\dagger} e^{-lH_c}\Lambda \begin{pmatrix} |D(2p){+}\rangle \\ |\overline{D}(2p){+}\rangle \end{pmatrix} . \tag{3.5.7}$$

In this formalism, the calculation of the cylinder diagram for an open string with CP factor Λ can be rewritten as the closed string exchange amplitude $\langle\langle\Lambda\rangle\rangle$. The amplitude for a closed string to be emitted and reabsorbed by the D(2p)+$\overline{\text{D}}$(2p)+ pair is equal to

$$\left\langle\left\langle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\rangle\right\rangle = \langle\langle I + \sigma_1 \rangle\rangle \ . \tag{3.5.8}$$

When we orbifold by projecting out σ_2 and σ_3, we see that this amplitude is unchanged. However, we know from our earlier discussion that the resulting object is a single D-brane. Therefore, we should be able to rewrite (3.5.8) as the emission and absorption of a closed string by a single $\widehat{\text{D(2p)}}$. Attempting this, we find

$$\langle\langle I + \sigma_1 \rangle\rangle \quad = \quad \begin{cases} 4 \int dl \, \langle \widehat{D(2p)} + | e^{-lH_c} | \widehat{D(2p)} + \rangle \\ 4 \int dl \, \langle \widehat{D(2p)} - | e^{-lH_c} | \widehat{D(2p)} - \rangle \, . \end{cases} \tag{3.5.9}$$

This amplitude can be written in terms of either a $\widehat{\text{D(2p)}}$+ or a $\widehat{\text{D(2p)}}$−, but our previous tachyon charge argument singles out the $\widehat{\text{D(2p)}}$+.

If we orbifold one more time by $(-1)^{F_L^s}$, the bulk transforms back to type 0A. The action of the orbifold on the D-brane's open string modes can be determined by examining the two-point functions of the theory. The existence of nonzero two-point functions between open strings on the D-brane and closed strings in the bulk allows us to determine the action of $(-1)^{F_L^s}$ on the open strings by requiring the correlation functions to be invariant. As in the type II case [14], the orbifold's effect on the $\widehat{\text{D(2p)}}$+ is to project out the open strings with CP factor σ_1. Removing the σ_1 from

(3.5.8) leaves the following amplitude for closed string emission and absorption:

$$\langle\!\langle I \rangle\!\rangle \quad = \quad \begin{cases} 2\int dl\, \langle D(2p)+|e^{-lH_c}|D(2p)+\rangle \\ 2\int dl\, \langle \overline{D}(2p)+|e^{-lH_c}|\overline{D}(2p)+\rangle \\ 2\int dl\, \langle D(2p)-|e^{-lH_c}|D(2p)-\rangle \\ 2\int dl\, \langle \overline{D}(2p)-|e^{-lH_c}|\overline{D}(2p)-\rangle\,. \end{cases}$$

This time, the amplitude can be written in four ways, in terms of a D(2p)+, $\overline{\text{D}}$(2p)+, D(2p)−, or $\overline{\text{D}}$(2p)−. Based on the previous tachyon charge argument, we can rule out the last two possibilities, so we know the resulting object is either a stable D(2p)+ or a stable $\overline{\text{D}}$(2p)+ in type 0A. This agrees with Sen's observation in [14] that there is an inherent ambiguity as to whether the resulting object is a brane or an anti-brane.

3.6 Tachyonic Kink

The other component to the descent relations is the tachyonic kink. As shown in figure 1, kinking one of the two tachyons on a Dp$\overline{\text{D}}$p in a type II theory yields a $\widehat{\text{D(p}-1)}$ in the same theory and kinking the remaining tachyon results in a D(p−2). This part of the descent relations is shown by taking a series of marginal deformations that connect the Dp$\overline{\text{D}}$p to the tachyonic kink and following what happens to the CFT under these deformations.

To outline the series of marginal deformations, we will use the D1$\overline{\text{D}}$1 pair in 0B for simplicity. The details of this analysis can be found in [14,30]. We begin with the D1$\overline{\text{D}}$1 pair wrapped on a circle of radius R and make the following deformations:

1. We increase the gauge field on the $\overline{\text{D}}$1 so that the open strings with CP factors σ_1 and σ_2 are antiperiodic around the compactification circle. In particular, the

tachyon field with CP factor σ_1 is moded by half-integers as

$$T(x,t) = \sum_{n \in \mathbf{Z}} T_{n+\frac{1}{2}}(t) e^{i(n+\frac{1}{2})\frac{x}{R}} \,. \tag{3.6.1}$$

2. The radius of the circle is taken down to $R = 1/\sqrt{2}$. At this value, the $T_{\pm\frac{1}{2}}$ modes are massless and, therefore, correspond to marginal deformations.

3. A vev of $-i$ is given to $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ which corresponds to

$$T(x) = \sin \frac{x}{2R} \,. \tag{3.6.2}$$

   This is the tachyonic kink.

4. The radius, R, is taken back to infinity.

Step number three will be our main focus. In order to understand the effect of this step, we first bosonize the worldsheet spinors ψ_L and ψ_R (often denoted as ψ and $\tilde{\psi}$) whose spacetime indices correspond to the compactified direction. In addition to ψ_L, ψ_R, and the corresponding X $(= X_L + X_R)$, we introduce four new spinors ξ_L, ξ_R, η_L, and η_R, and two new bosons, ϕ $(= \phi_L + \phi_R)$ and ϕ' $(= \phi'_L + \phi'_R)$. The bosonization equations relating them are

$$e^{\pm i\sqrt{2}X_L} \sim (\xi_L \pm i\eta_L) \,, \tag{3.6.3}$$

$$e^{\pm i\sqrt{2}\phi_L} \sim (\xi_L \pm i\psi_L) \,, \tag{3.6.4}$$

$$e^{\pm i\sqrt{2}\phi'_L} \sim (\eta_L \pm i\psi_L) \,, \tag{3.6.5}$$

and similarly for the right-moving fields. We also have the relations

$$\xi_L\eta_L \sim \partial X_L \,, \quad \xi_L\psi_L \sim \partial\phi_L \,, \quad \eta_L\psi_L \sim \partial\phi'_L \,, \tag{3.6.6}$$

as well as the corresponding right-moving relations. Remember, these fields are specifically those fields whose spacetime indices correspond to the compactified direction. Written in terms of the new bosonic field, the tachyonic kink is made by inserting

$$\exp\left(i\frac{\sigma_1}{2\sqrt{2}}\oint\partial\phi\right) \tag{3.6.7}$$

at the boundary of the disk. In step four, the radius is taken back to infinity by inserting vertex operators of the form $\partial X\overline{\partial}X$. When the contour integral of $\partial\phi$ is contracted around each of these operators, they are converted into $-\partial\phi'\overline{\partial}\phi'$. This corresponds to decreasing the ϕ' radius, so we must introduce a T-dual variable, ϕ'' related to the ϕ' as

$$\phi''_L = \phi'_L\,, \quad \phi''_R = -\phi'_R\,, \quad R_{\phi''} = 1/R_{\phi'}\,. \tag{3.6.8}$$

This converts the Neumann boundary condition on ϕ' to a Dirichlet boundary condition on ϕ'' and we are left with a D0-brane where ϕ'' is the new spacetime coordinate in place of X.

This process is easily extended to Dp$\overline{\text{D}}$p pairs for p other than 1 since the other worldsheet fields are left unchanged. This is, in fact, the key to understanding whether a Dp+$\overline{\text{D}}$p+ gets kinked to a $\widehat{\text{D(p}-1)}$+ or a $\widehat{\text{D(p}-1)}-$. Let us take a look now at what the + and − correspond to in terms of worldsheet fields. The boundary state $|Dp,\eta\rangle$ satisfies the following equations:

$$\begin{aligned}
\partial_n X^\mu |Dp,\eta\rangle &= 0\,, \quad \mu = 0,\ldots,p \\
(X^i - y^i)|Dp,\eta\rangle &= 0\,, \quad i = p+1,\ldots,9 \\
(\psi^\mu - \eta\tilde{\psi}^\mu)|Dp,\eta\rangle &= 0\,, \quad \mu = 0,\ldots,p \\
(\psi^i + \eta\tilde{\psi}^i)|Dp,\eta\rangle &= 0\,, \quad i = p+1,\ldots,9 \\
(b - \tilde{b})|Dp,\eta\rangle &= 0\,, \\
(c - \tilde{c})|Dp,\eta\rangle &= 0\,, \\
(\gamma - \eta\tilde{\gamma})|Dp,\eta\rangle &= 0\,, \\
(\beta - \eta\tilde{\beta})|Dp,\eta\rangle &= 0\,.
\end{aligned} \tag{3.6.9}$$

The first four of these equations are the familiar boundary conditions on the matter fields. The last four can be obtained by demanding BRST invariance of the boundary state [29].

The only worldsheet fields that are affected by the kink are those whose spacetime index is the same as the compactified direction. For example, no matter what tachyonic kinking procedure we can imagine, ψ^0 will certainly be unaffected. Since the η value of the $|Dp,\eta\rangle$ D-brane can be read off from the boundary condition on ψ^0, η is invariant under all marginal deformations corresponding to tachyonic kinks. This means that a Dp+$\overline{\mathrm{D}}$p+ gets kinked to a $\widehat{\mathrm{D(p{-}1)}}$+.

Now we claim that the rest of the kink analysis goes through the same as it did in the case of the type II theories. How can we be so sure of this? The type 0 and type II theories differ in their perturbative closed string spectra, but the marginal deformations needed to bring about a tachyonic kink uses only those parts of the closed string spectra that type 0 and type II have in common. In particular, the only closed string vev that is deformed is that of the graviton which can be found in the (NS+,NS+) sector of all type 0 and type II theories. All other deformations have to do with open strings, and the bosonic open string spectra on D-branes in type 0 and type II theories are identical. This can be seen by comparing tables 1 and 2 with tables 4 and 5 in Appendix A.

Let us check that the Dp+$\overline{\mathrm{D}}$p+ gets kinked to the $\widehat{\mathrm{D(p-1)}}$+ by considering the amplitude for the emission of a closed string tachyon. From table 3 and equation (3.5.3), we see that the combined D1+$\overline{\mathrm{D}}$1+ pair in type 0B has a nonzero tachyon charge (Recall that $\eta = q\bar{q}$). The amplitude under consideration is the closed tachyon tadpole amplitude: a disk with the tachyon vertex operator inserted in the bulk. Again, kinematics force the momentum of the emitted tachyon to be perpendicular to the D1+$\overline{\mathrm{D}}$1+ pair. Therefore, there are no potential contractions between the tachyon vertex operator, $e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}$, and the tachyonic kink operator in (3.6.7). The sign of the amplitude is not changed by the marginal deformations, so the result is a $\widehat{\mathrm{D0}}$ brane that couples to the closed tachyon with the same sign as the D1+$\overline{\mathrm{D}}$1+, namely a $\widehat{\mathrm{D0}}$+.

The result we have established here for the D1+$\overline{\mathrm{D}}$1+ pair in type 0B can easily be extended to all Dp+$\overline{\mathrm{D}}$p+ pairs and Dp−$\overline{\mathrm{D}}$p− pairs for p even in 0A and p odd in 0B. The tachyonic kink on an unstable $\widehat{\mathrm{Dp}}$+ or $\widehat{\mathrm{Dp}}$−, for p > 0, can be analyzed by the following procedure [14]. Take the unstable $\widehat{\mathrm{D1}}$+ in 0A as an example. If we T-dualize the D1+$\overline{\mathrm{D}}$1+ pair in type 0B, we find that the D0+$\overline{\mathrm{D}}$0+ pair in 0A is connected by marginal deformations to the $\widehat{\mathrm{D1}}$+ in 0A. By running the marginal deformations backwards, we see that the D0+$\overline{\mathrm{D}}$0+ corresponds to a kink-antikink pair on the $\widehat{\mathrm{D1}}$+. This allows us to identify the tachyonic kink on the $\widehat{\mathrm{D1}}$+ as a stable D0+ in type 0A. The flowchart of descent relations in the type 0 theories is given in figure 2.

$$
\begin{array}{cccccccc}
 & & & & & \downarrow & & \downarrow \\
 & & & \text{0B D(2p+1)+}\overline{\text{D}}\text{(2p+1)+} & \rightarrow & \text{0A }\widehat{\text{D(2p+1)}}\text{+} & \rightarrow & \text{0B D(2p+1)+} \\
 & & & \downarrow & & \downarrow & & \\
 & \text{0A D(2p)+}\overline{\text{D}}\text{(2p)+} & \rightarrow & \text{0B }\widehat{\text{D(2p)}}\text{+} & \rightarrow & \text{0A D(2p)+} & & \\
 & \downarrow & & \downarrow & & & & \\
\rightarrow & \text{0A }\widehat{\text{D(2p}-\text{1)}}\text{+} & \rightarrow & \text{0B D(2p}-\text{1)+} & & & & \\
 & \downarrow & & & & & & \\
\rightarrow & \text{0A D(2p}-\text{2)+} & & & & & &
\end{array}
$$

Figure 2: Descent relations for the type 0 theories. Horizontal arrows denote modding by $(-1)^{F_L^s}$. Vertical arrows denote the tachyonic kink. A similar diagram exists with $+ \rightarrow -$.

3.7 $|Dp, \eta\rangle$: $\eta = +1$ vs. $\eta = -1$

It is important to stress that the value of η in $|Dp, \eta\rangle$ does not just affect the R-R charges of the D-brane. It has an important effect on many string amplitudes. In fact, we will be able to show below that Dp+ and Dp− branes have the same tadpole couplings to all (NS+,NS+) fields and opposite tadpole couplings to all (NS−,NS−) fields.

Let us first try to see the opposite tachyon charges of the Dp+ and Dp− at the level of a string calculation. Emission of a tachyon from a D-brane in a type 0 theory is given by a disk amplitude with the tachyon vertex operator in the bulk and appropriate boundary conditions on the edge. Note from (3.6.9) that these boundary conditions depend on η. Equations (3.6.9) are in terms of the fields defined on the upper half plane, so once we map our tachyon amplitude to the upper half plane, the following η-dependent equations must hold on the real axis:

$$\tilde{\psi}^\mu = \eta\psi^\mu\,, \qquad \tilde{\psi}^i = -\eta\psi^i\,, \tag{3.7.1}$$

$$\tilde{\gamma} = \eta\gamma\,, \qquad \tilde{\beta} = \eta\beta\,. \tag{3.7.2}$$

The doubling trick [31] extends the string calculation to the entire complex plane by defining

$$\tilde{\psi}^\mu(\overline{z}) = \eta\psi^\mu(\overline{z})\,, \qquad \tilde{\psi}^i(\overline{z}) = -\eta\psi^i(\overline{z})\,, \tag{3.7.3}$$

$$\tilde{\gamma}(\overline{z}) = \eta\gamma(\overline{z})\,, \qquad \tilde{\beta}(\overline{z}) = \eta\beta(\overline{z}) \tag{3.7.4}$$

on the lower half plane. In actual calculations, β and γ are rebosonized in terms of the free bosons Φ and χ as

$$\beta \cong e^{-\Phi+\chi}\partial\chi\,, \qquad \gamma \cong e^{\Phi-\chi}\,. \tag{3.7.5}$$

The doubling trick identifications on γ and β can be rewritten as

$$\begin{aligned} \tilde{\Phi}(\overline{z}) &= \Phi(\overline{z}) + \frac{i\pi}{2}(1-\eta)\,, \\ \tilde{\chi}(\overline{z}) &= \chi(\overline{z})\,. \end{aligned} \tag{3.7.6}$$

After mapping to the upper half plane and then using the doubling trick, the amplitude has become

$$\langle e^{-\Phi(z)-\Phi(\overline{z})-i\pi(1-\eta)/2}e^{-ik\cdot X}\rangle = (-1)^{(1-\eta)/2}\langle e^{-\Phi(z)-\Phi(\overline{z})}e^{-ik\cdot X}\rangle\,. \tag{3.7.7}$$

Here we see the explicit dependence on η of the D-brane's tachyon charge.

A somewhat complicated, but instructive, example is to look at C goes to $\overline{C}$ scattering as depicted in figure 3, where C and $\overline{C}$ are massless bosons from the two

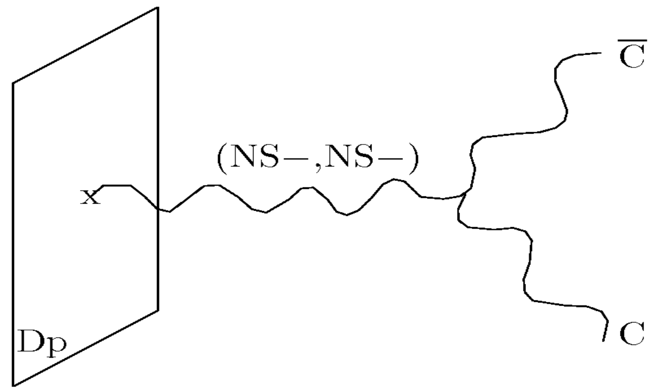


Figure 3

different R-R sectors.

The D-brane in type 0 theories couples to (NS−,NS−) closed strings and there are vertices in the low energy spacetime action that connect (NS−,NS−) strings to a C and a $\overline{C}$ [26]. The string diagram that contributes to this process is a disk with V_C and $V_{\overline{C}}$ operators. These massless R-R vertex operators are given by

$$
\begin{aligned}
V_i^{C_{m-1}}(z_i,\bar{z}_i) &= (P_-\Gamma_{i(m)})^{AB} : V_{-1/2\,A}(p_i,z_i) :: \tilde{V}_{-1/2\,B}(p_i,\tilde{z}_i) :, \qquad (3.7.8)\\
V_i^{\overline{C}_{m-1}}(z_i,\bar{z}_i) &= (P_+\Gamma_{i(m)})^{AB} : V_{-1/2\,A}(p_i,z_i) :: \tilde{V}_{-1/2\,B}(p_i,\tilde{z}_i) :, \qquad (3.7.9)
\end{aligned}
$$

where we are using the notation of [32]. The objects in these vertex operators are defined as

$$
\begin{aligned}
V_{-1/2\,A}(p_i,z_i) &= e^{-\Phi(z_i)/2} S_A(z_i) e^{ip_i\cdot X_L(z_i)}\,, \qquad (3.7.10)\\
P_\pm &= (1\pm\gamma_{11})/2\,, \qquad (3.7.11)\\
\Gamma_{(n)} &= \frac{a_n}{n!} F_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n}\,, \qquad (3.7.12)
\end{aligned}
$$

where S_A is the spin field, $\gamma_{11}=\gamma^0\ldots\gamma^9$, and $F_n = dC_{n-1}$.

Under the doubling trick, the spin field $\tilde{S}_A$ will be identified as

$$\tilde{S}_A(\bar{z}) = M_A{}^B S_B(\bar{z})\,, \qquad (3.7.13)$$

for some matrix M. This matrix can be specified [32] by considering the following OPE's.

$$\psi^\mu(z)S_A(w) \sim (z-w)^{-1/2}\frac{1}{\sqrt{2}}(\gamma^\mu)_A{}^B S_B(w) + \ldots \tag{3.7.14}$$

$$\tilde{\psi}^\mu(\overline{z})\tilde{S}_A(\overline{w}) \sim (\overline{z}-\overline{w})^{-1/2}\frac{1}{\sqrt{2}}(\gamma^\mu)_A{}^B \tilde{S}_B(\overline{w}) + \ldots \tag{3.7.15}$$

The doubling trick identification for $\tilde{\psi}^\mu$ is $\tilde{\psi}^\mu(\overline{z}) = \eta D^\mu{}_\nu \psi^\nu(\overline{z})$, where $D^\mu{}_\nu = (\delta^\alpha{}_\beta, -\delta^i{}_j)$. In order for (3.7.15) to be consistent with (3.7.14), M must satisfy

$$(\gamma^\mu)_A{}^B = D^\mu{}_\nu (M^{-1}\gamma^\nu M)_A{}^B\,. \tag{3.7.16}$$

This can be rewritten as $(M\gamma^\mu) = D^\mu{}_\nu(\gamma^\nu M)$ which implies that M is of the form

$$M = \begin{cases} a\gamma^0\ldots\gamma^p & \text{for p+1 odd, } \eta = 1 \\ b\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 even, } \eta = 1 \\ c\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 odd, } \eta = -1 \\ d\gamma^0\ldots\gamma^p & \text{for p+1 even, } \eta = -1\,. \end{cases} \tag{3.7.17}$$

To fix the phases, the OPE's

$$S_A(z)S_B(w) \sim (z-w)^{-5/4}C^{-1}_{AB} + \ldots \tag{3.7.18}$$

$$\tilde{S}_A(\overline{z})\tilde{S}_B(\overline{w}) \sim (\overline{z}-\overline{w})^{-5/4}C^{-1}_{AB} + \ldots \tag{3.7.19}$$

are used to find that $M^{-1} = C^{-1}M^TC$. Since all the γ^μ and γ_{11} anticommute with

C, we find the phases up to an overall sign:

$$M = \begin{cases} \pm i\gamma^0 \dots \gamma^p & \text{for p+1 odd, } \eta = 1 \\ \pm \gamma^0 \dots \gamma^p \gamma_{11} & \text{for p+1 even, } \eta = 1 \\ \pm \gamma^0 \dots \gamma^p \gamma_{11} & \text{for p+1 odd, } \eta = -1 \\ \pm i\gamma^0 \dots \gamma^p & \text{for p+1 even, } \eta = -1\,. \end{cases} \tag{3.7.20}$$

From now on, we will write M as M_η to distinguish between the two forms it takes for fixed p. Equation (3.7.20) gives the relationships between M_+ and M_- as

$$M_- = \pm i M_+ \gamma_{11}\,. \tag{3.7.21}$$

The amplitude for $C \to \overline{C}$ scattering off a Dp+ is [33]

$$\begin{aligned} A(C,\overline{C})_+ = &-\frac{i\kappa^2 T_p}{2}[\frac{1}{2}\mathrm{Tr}(P_-\Gamma_{1(m)}M_+\gamma^\mu)\mathrm{Tr}(P_+\Gamma_{2(n)}M_+\gamma_\mu)B(-t/2+1/2,-2s) \\ &- \mathrm{Tr}(P_-\Gamma_{1(m)}C^{-1}\Gamma^T_{2(n)}C)B(-t/2-1/2,-2s+1) \\ &- \mathrm{Tr}(P_-\Gamma_{1(m)}M_+\Gamma_{2(n)}M_+)B(-t/2+1/2,-2s+1)]\,. \end{aligned} \tag{3.7.22}$$

Since the Euler beta function is defined as

$$B(a,b) = \int_0^1 dy\, y^{a-1}(1-y)^{b-1}\,, \tag{3.7.23}$$

we see that the poles in the t channel are $m^2 = (4n-2)/\alpha'$ for $n = 0, 1, \dots$. These poles correspond to the masses of the closed strings in the (NS−,NS−) sector.

To obtain $A(C,\overline{C})_-$, the amplitude for $C \to \overline{C}$ scattering off a Dp−, from $A(C,\overline{C})_+$, we must replace M_+ with M_- and $e^{-\Phi(\overline{z})/2}$ with $e^{-\Phi(\overline{z})/2-i\pi/2}$. It is simple to check that the amplitude is invariant under replacing M_+ with M_-. In the correlation function, there are two factors of $e^{-\Phi(\overline{z})/2}$ coming from the two R-R vertex operators.

After replacing them with $e^{-\Phi(\bar{z})/2-i\pi/2}$, each one contributes a factor of i for a total phase of -1. In summary, we find that

$$A(C,\overline{C})_{+} = -A(C,\overline{C})_{-}\,. \tag{3.7.24}$$

This shows that the Dp+ and Dp− couple with opposite signs to all (NS−,NS−) fields.

How can this phenomenon be understood in a more direct manner? Consider the tadpole amplitude for emission of a closed string from a D-brane. If the closed string is in one of the NS-NS sectors, the amplitude is a disk with the closed string vertex operator in the (−1,−1) picture. For a NS-NS string, the amplitude for emission from a Dp+ can be converted into an amplitude for emission from a Dp− by multiplying by -1 for each factor of $e^{-\tilde{\Phi}}$ and $\tilde{\psi}^{\mu}$. In the (−1,−1) picture, the NS-NS vertex operator has as many $\tilde{\psi}$'s as does the corresponding Fock state. Therefore, the Dp− amplitude differs from the Dp+ amplitude by a factor of $(-1)^{\tilde{F}}$, where $\tilde{F}$ is the right-moving worldsheet fermion number of the NS-NS closed string state. In other words, Dp+ and Dp− have the same tadpole couplings to all (NS+,NS+) fields and opposite tadpole couplings to all (NS−,NS−) fields.

It is clear how to generalize this to a general disk amplitude on a D-brane. To convert a general disk amplitude for a D+ into the same amplitude with a D−, we multiply by -1 for each $e^{-\tilde{\Phi}}$ and $\tilde{\psi}^{\mu}$, and we replace M_{+} with $\pm iM_{+}\gamma_{11}$ for each spin field. Since a fermionic state can not transform into a bosonic one, the number of M_{η}'s will be even in any nonzero amplitude, so the sign ambiguity in that replacement is insignificant.

3.8 Summary

We set out to find the descent relations for the type 0 theories. We found that we must start with either a D+$\overline{\text{D}}$+ pair or a D−$\overline{\text{D}}$− pair and that the + and − are invariant under the orbifold and kink operations. This means we have two copies of the usual descent relation chain for the type 0 theories: one for D+ branes and one for D− branes. We then asked why we should care about the distinction between a D+ brane and a D− brane. While it is fairly well known that the stable D+ and D− have the same coupling to half of the massless R-R fields and equal and opposite couplings to the other half, we have shown that the D+ and D− have the same tadpole couplings to half of the NS-NS fields and equal and opposite tadpole couplings to the other half.

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3.9 Appendix A. Open String Spectrum

In this appendix, we will find the open string spectrum on type II and type 0 D-branes. We begin by considering the closed string exchange amplitudes between boundary states, which are given in [34]. Motivated by the usual worldsheet duality of the cylinder diagram, this result can be converted into an open string loop amplitude.

The results are as follows:

$$
\begin{aligned}
\int dl\ {}_{\text{NS-NS}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{NS-NS}} &= \int \tfrac{dt}{2t}\mathrm{Tr}_{\text{NS}}[e^{-tH_{\text{open}}}] \\
\int dl\ {}_{\text{NS-NS}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,-\eta\rangle_{\text{NS-NS}} &= -\int \tfrac{dt}{2t}\mathrm{Tr}_{\text{R}}[e^{-tH_{\text{open}}}] \\
\int dl\ {}_{\text{R-R}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{R-R}} &= \int \tfrac{dt}{2t}\mathrm{Tr}_{\text{NS}}[(-1)^F e^{-tH_{\text{open}}}] \\
\int dl\ {}_{\text{R-R}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,-\eta\rangle_{\text{R-R}} &= -\int \tfrac{dt}{2t}\mathrm{Tr}_{\text{R}}[(-1)^F e^{-tH_{\text{open}}}]
\end{aligned}
\tag{3.9.1}
$$

We will combine equations (3.9.1) with the expressions [25] for the type II D-branes in terms of boundary states,

$$
\left.
\begin{aligned}
|Dp\rangle &= (|Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}) \\
&\qquad +(|Bp,+\rangle_{\text{R-R}} + |Bp,-\rangle_{\text{R-R}}) \\
|\overline{D}p\rangle &= (|Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}) \\
&\qquad -(|Bp,+\rangle_{\text{R-R}} + |Bp,-\rangle_{\text{R-R}})
\end{aligned}
\right\}\ \text{for p even (odd) in IIA (IIB)}
\tag{3.9.2}
$$

$$
\left.
|\widehat{D}p\rangle = |Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}
\right\}\ \text{for all p in IIA and IIB}
\tag{3.9.3}
$$

and the expressions for the type 0 D-branes in terms of boundary states,

$$
\left.
\begin{aligned}
|Dp,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} + |Bp,+\rangle_{\text{R-R}} \\
|Dp,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} + |Bp,-\rangle_{\text{R-R}} \\
|\overline{D}p,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} - |Bp,+\rangle_{\text{R-R}} \\
|\overline{D}p,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{R-R}}
\end{aligned}
\right\}\ \text{for p even (odd) in 0A (0B)}
\tag{3.9.4}
$$

$$
\left.
\begin{aligned}
|\widehat{D}p,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} \\
|\widehat{D}p,-\rangle &= |Bp,-\rangle_{\text{NS-NS}}
\end{aligned}
\right\}\ \text{for all p in 0A and 0B}\,.
\tag{3.9.5}
$$

It is impossible for a R-R string to spontaneously convert into a NS-NS string, or vice versa, so we know that

$$
{}_{\text{NS-NS}}\langle Bp,\eta'|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{R-R}} = 0\,.
\tag{3.9.6}
$$

Now, to find the spectrum on open strings beginning and ending on a stable Dp+ in the type 0 theories, we will rewrite the closed string exchange diagram as a trace over open string states. We have everything we need to perform this calculation; combining equations (3.9.1) and (3.9.4), we find

$$
\begin{aligned}
& \int dl \, \langle Dp, +| e^{-lH_{\text{closed}}} | Dp, + \rangle \\
= & \int dl \; {}_{\text{NS-NS}}\langle Bp, +| e^{-lH_{\text{closed}}} | Bp, + \rangle_{\text{NS-NS}} + \int dl \; {}_{\text{R-R}}\langle Bp, +| e^{-lH_{\text{closed}}} | Bp, + \rangle_{\text{R-R}} \\
= & \int \frac{dt}{2t} \text{Tr}_{\text{NS}}[e^{-tH_{\text{open}}}] + \int \frac{dt}{2t} \text{Tr}_{\text{NS}}[(-1)^F e^{-tH_{\text{open}}}] \\
= & \int \frac{dt}{2t} \text{Tr}_{\text{NS}}[(1 + (-1)^F) e^{-tH_{\text{open}}}] \\
= & \int \frac{dt}{t} \text{Tr}_{\text{NS+}}[e^{-tH_{\text{open}}}] \, .
\end{aligned} \tag{3.9.7}
$$

So we see that the open strings beginning and ending on a stable Dp+ in the type 0 theories are NS+. Proceeding in this manner, we can find the spectrum of open strings on all possible combinations of D-branes in the type 0 and type II theories. The full results for the type 0 theories are given in tables 1 and 2 in section 3.3. The results for the type II theories are given in tables 4 and 5 below.

Open Spectrum on Stable D-branes (p odd in IIB, p even in IIA)		
$\sigma = 0$	$\sigma = \pi$	Spectrum
Dp	Dp	NS+, R−
Dp	$\overline{\text{D}}$p	NS−, R+

Table 4: The other two cases obtained by the following operation under which the spectrum is invariant: D $\leftrightarrow$ $\overline{\text{D}}$.

Open Spectrum on Unstable D-branes (all p in IIA and IIB)		
$\sigma = 0$	$\sigma = \pi$	Spectrum
$\widehat{\mathrm{Dp}}$	$\widehat{\mathrm{Dp}}$	NS+, NS−, R+, R−

Table 5

3.10 Appendix B. Orbifold of 0A/0B

The action of $(-1)^{F_L^s}$ can be represented as a 2π spacetime rotation on the left-movers. Under this rotation, the left-sector bosons (NS) are invariant and the left-sector fermions (R) pick up a minus sign. We can pick any spatial plane for this rotation and for our purposes here we select the 8-9 plane.

The situation is greatly simplified if we use complexified coordinates [24] for those left-moving fields whose indices are in the 8-9 plane,

$$\begin{aligned} \Psi^4 &= \frac{1}{\sqrt{2}}(\psi^8 + i\psi^9)\,, \\ \Psi^{\bar{4}} &= \frac{1}{\sqrt{2}}(\psi^8 - i\psi^9)\,, \end{aligned} \tag{3.10.1}$$

$$\begin{aligned} \partial Z^4 &= \frac{1}{\sqrt{2}}(\partial X^8 + i\partial X^9)\,, \\ \partial Z^{\bar{4}} &= \frac{1}{\sqrt{2}}(\partial X^8 - i\partial X^9)\,. \end{aligned} \tag{3.10.2}$$

With this notation, a rotation on the left-movers by angle θ in the 8-9 plane has the following action on the fields:

$$\begin{aligned}
\Psi^4 &\rightarrow e^{i\theta}\Psi^4\,, \\
\Psi^{\overline{4}} &\rightarrow e^{-i\theta}\Psi^{\overline{4}}\,, \qquad (3.10.3) \\
\partial Z^4 &\rightarrow e^{i\theta}\partial Z^4\,, \\
\partial Z^{\overline{4}} &\rightarrow e^{-i\theta}\partial Z^{\overline{4}}\,. \qquad (3.10.4)
\end{aligned}$$

We wish to find the orbifold of type 0A by $(-1)^{F^s_L}$. This is an asymmetric, abelian orbifold with group elements $\{1,\,(-1)^{F^s_L}\}$. The untwisted sector, corresponding to the identity element, is simply the projection of 0A on states invariant under $(-1)^{F^s_L}$. It is clear that the invariant states are those in the sectors (NS+,NS+) and (NS−,NS−). Let us check that we get the same result by representing $(-1)^{F^s_L}$ as a rotation by 2π on the left-movers. On the NS sector ground state vertex operator, $1 \rightarrow 1$; the NS sector is invariant. To consider the action on the R sector ground state vertex operator, we must bosonize the complexified fermions as

$$\begin{aligned}
\Psi^4 &= e^{iH^4}\,, \\
\Psi^{\overline{4}} &= e^{-iH^4}\,, \qquad (3.10.5)
\end{aligned}$$

and likewise for the other fermions. In terms of these bosonic H fields, the spin operator takes the form

$$\Theta_s = e^{\,i\sum\limits_{a=1}^{4} s_a H^a}\,, \qquad (3.10.6)$$

where the $s_a = \pm 1/2$. Since Ψ^4 transforms under the $\theta = 2\pi$ rotation as (3.10.3), $\exp(\frac{1}{2}iH^4)$ transforms as

$$e^{\frac{1}{2}iH^4} \rightarrow e^{i\pi}e^{\frac{1}{2}iH^4} = -e^{\frac{1}{2}iH^4}\,. \qquad (3.10.7)$$

Therefore, the spin field, and subsequently the left-moving R sector vertex operator, picks up a minus sign from the 2π rotation; the (R+,R−) and (R−,R+) sectors are projected out.

In the twisted sector, the boundary conditions on the ∂Z^4 and Ψ^4 fields are as follows:

$$\begin{aligned} \partial Z^4(\sigma+2\pi) &= e^{2\pi i}\partial Z^4(\sigma)\,, \\ \partial Z^{\bar{4}}(\sigma+2\pi) &= e^{-2\pi i}\partial Z^{\bar{4}}(\sigma)\,, \end{aligned} \tag{3.10.8}$$

$$\begin{aligned} \Psi^4(\sigma+2\pi) &= e^{2\pi i(\beta+\nu)}\Psi^4(\sigma)\,, \\ \Psi^{\bar{4}}(\sigma+2\pi) &= e^{-2\pi i(\beta+\nu)}\Psi^{\bar{4}}(\sigma)\,, \end{aligned} \tag{3.10.9}$$

where $\nu = 0$ for R, $\nu = 1/2$ for NS, and $\beta = 1$. At first glance, it appears as though the boundary conditions are unchanged. However, if we continuously change the boundary condition factor $\exp(2\pi i\beta)$ from $\beta = 0$ to $\beta = 1$, we see that the moding of the Fourier coefficients has changed from n for both ∂Z^4 and $\partial Z^{\bar{4}}$ and $n+\nu$ for both Ψ^4 and $\Psi^{\bar{4}}$ to

$$\begin{aligned} \alpha^4 &: \; n+1\,, \\ \alpha^{\bar{4}} &: \; n-1\,, \\ \Psi^4 &: \; n+1+\nu\,, \\ \Psi^{\bar{4}} &: \; n-1-\nu\,. \end{aligned} \tag{3.10.10}$$

This phenomenon, known as spectral flow, has an important consequence for the ground state of the theory. When we began with $\beta = 0$, the ground state was defined as

$$\Psi^4_{n+\nu}|0\rangle = \Psi^{\bar{4}}_{n+1-\nu}|0\rangle = 0 \quad \text{for } n = 0, 1, \ldots\,, \tag{3.10.11}$$

with similar equations for the other Ψ. The effect of continuously changing β from 0 to 1 is that we replace ν with $\nu+1$ in these equations. The ground state now satisfies the conditions

$$\Psi^4_{n+\nu+1}|0\rangle = \Psi^{\bar{4}}_{n-\nu}|0\rangle = 0 \quad \text{for } n = 0, 1, \ldots \,. \tag{3.10.12}$$

The $|0\rangle$ state is no longer the ground state because $\Psi^4_\nu|0\rangle \neq 0$ and $\Psi^{\bar{4}}_{-\nu}|0\rangle = 0$. The true ground state is

$$|0\rangle' = \Psi^4_\nu|0\rangle \tag{3.10.13}$$

since

$$\Psi^4_\nu|0\rangle' = \Psi^4_\nu\Psi^4_\nu|0\rangle = 0 \tag{3.10.14}$$

and

$$\Psi^{\bar{4}}_{-\nu}|0\rangle' = \Psi^{\bar{4}}_{-\nu}\Psi^4_\nu|0\rangle = \{\Psi^{\bar{4}}_{-\nu}, \Psi^4_\nu\}|0\rangle = |0\rangle \neq 0\,. \tag{3.10.15}$$

However, now the GSO condition on the left-movers,

$$(-1)^F|0\rangle = \pm|0\rangle \tag{3.10.16}$$

has become

$$(-1)^F|0\rangle' = -\Psi^4_\nu(-1)^F|0\rangle = \mp|0\rangle'\,. \tag{3.10.17}$$

We see that the GSO conditions on the left-movers has been reversed.

This leaves us with the following twisted sector:

$$(\text{NS}-,\text{NS}+) \quad (\text{NS}+,\text{NS}-) \quad (\text{R}-,\text{R}-) \quad (\text{R}+,\text{R}+)\,. \tag{3.10.18}$$

Of these four groups of states, we keep only those that will combine with the untwisted sector to give us a modular invariant theory. For abelian orbifolds, the correct criteria for the twisted states to ensure modular invariance is level matching. In the (NS−,NS+) and (NS+,NS−) sectors, there is no way to obtain $L_0 = \tilde{L}_0$, so we drop these states.

In the end, we are left with the (NS+,NS+) and (NS−,NS−) states from the untwisted sector and the (R−,R−) and (R+,R+) states from the twisted sector. Combined, these give the spectrum of the type 0B theory as given in (3.2.2). The argument works in the same way to get type 0A from 0B.

Chapter 4

AdS Solutions of 2D Type 0A

4.1 Introduction

AdS backgrounds of string theory are often fruitful arenas for studying holographic dualities and for constructing sigma models with R-R fluxes, among other things. AdS backgrounds of type 0A string theory should be no exception. In particular, the recent discovery of a matrix quantum mechanics dual to two-dimensional type 0A string theory [35, 36] suggests a promising direction for understanding AdS_2/CFT_1 [13]. Also, two-dimensional AdS presents one of the simplest backgrounds in which to study sigma models in R-R flux.

As a first step in these pursuits, we present here a two-parameter family of AdS_2 solutions to the two-dimensional type 0A effective action. One of the parameters is the tachyon field T (equivalently, the ratio of dualized R-R field strengths, q_+^2/q_-^2). The other parameter is the string coupling $e^{2\Phi}$ (equivalently, the magnitude of the field strength q^2). In these solutions, string loops can be suppressed by reducing the string coupling, but the high curvature of the spaces makes higher order α' corrections important.

In section 4.2, we briefly review the 2D 0A string theory. In section 4.3, we present our family of AdS_2 solutions. In section 4.4, we discuss possible corrections to our solutions from higher order terms in the effective action.

4.2 Two-Dimensional Type 0A

The ten-dimensional type 0A string theory is given by the same worldsheet action as the type IIA string, but with a GSO projection onto the closed string sectors

$$(\mathrm{NS}+,\mathrm{NS}+) \quad (\mathrm{NS}-,\mathrm{NS}-) \quad (\mathrm{R}+,\mathrm{R}-) \quad (\mathrm{R}-,\mathrm{R}+) \tag{4.2.1}$$

where $+$ and $-$ denote the eigenvalue of the worldsheet fermion number operator $(-1)^F$. In ten dimensions, each of these sectors contains a tower of states corresponding to the possible transverse oscillations. In two dimensions, however, there is no room for transverse oscillations, so the situation is much simpler. We have the graviton $g_{\mu\nu}$ and dilaton Φ in the (NS+,NS+) sector, the tachyon T in the (NS−,NS−) sector, and two gauge fields $C_\mu^{(\pm)}$ from the R-R sectors that give rise to two field strengths, $F_{\mu\nu}^{(\pm)}$. An allowed background for this theory is the two-dimensional linear dilaton vacuum ($g_{\mu\nu} = \eta_{\mu\nu}$ and $\Phi = \sqrt{\frac{2}{\alpha'}}\phi$) with an exponential tachyon wall ($T = \mu e^{\sqrt{\frac{2}{\alpha'}}\phi}$) and zero field strengths ($F_{\mu\nu}^{(\pm)} = 0$). The worldsheet action for this string theory is the action of $\mathcal{N} = 1$ super Liouville theory plus the action for a free scalar superfield.

The action for $\mathcal{N} = 1$ super Liouville theory can be written in superfield formalism[1] as

$$S_{\mathrm{SLT}} = \frac{1}{4\pi}\int d^2z d^2\theta \left(D\Phi\overline{D}\Phi + 2i\mu e^{b\Phi}\right) , \tag{4.2.2}$$

[1]Unfortunately, it is standard in the literature to use Φ for both the Liouville superfield and the background dilaton.

where Φ is the scalar superfield

$$\Phi = \sqrt{\frac{2}{\alpha'}}\phi + i\theta\psi + i\overline{\theta\psi} + i\theta\overline{\theta}F\,, \tag{4.2.3}$$

and the covariant derivatives are given by

$$D = \partial_\theta + \theta\partial_z\,, \quad \overline{D} = \partial_{\overline{\theta}} + \overline{\theta}\partial_{\overline{z}}\,. \tag{4.2.4}$$

In the case $b = 1$, this yields a theory with central charge $\hat{c} = 9$. When combined with the $\hat{c} = 1$ theory of a free scalar superfield X with action

$$S_{\mathrm{X}} = \frac{1}{4\pi}\int d^2z d^2\theta \left(DX\overline{D}X\right)\,, \tag{4.2.5}$$

we get a critical SCFT with central charge $\hat{c} = 10$. This is the worldsheet action for two-dimensional type 0A in the linear dilaton vacuum.

The effective spacetime action was calculated in [26] and was found to be

$$\int d^2x\sqrt{-g}\left[\frac{e^{-2\Phi}}{2\kappa^2}\left(\frac{8}{\alpha'} + R + 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) + \dots\right)\right.$$
$$\left. - \frac{\pi\alpha'}{2}f_3(T)\left(F^{(+)}\right)^2 - \frac{\pi\alpha'}{2}f_3(-T)\left(F^{(-)}\right)^2 + \dots\right]\,. \tag{4.2.6}$$

The first few terms in a Taylor expansion of the f_i functions are

$$f_1(T) = \frac{1}{2} + \dots\,, \qquad f_2(T) = \frac{1}{\alpha'}T^2 + \dots\,, \qquad f_3(T) = 1 - 2T + 2T^2 + \dots\,. \tag{4.2.7}$$

There is evidence that the exact expression for $f_3(T)$ is e^{-2T} [37,38], and we will use this form for f_3 in our calculations.

4.3 AdS$_2$ Solutions

4.3.1 Equations of Motion

To simplify the action (4.2.6), we will dualize the R-R field strengths:

$$
\begin{aligned}
-\frac{2\pi\alpha'}{4} f_3(\pm T)\left(F^{(\pm)}\right)^2 * 1 \quad &= \quad -\pi\alpha' f_3(\pm T) F^{(\pm)} \wedge * F^{(\pm)} \\
&\longrightarrow \quad -\frac{1}{4\pi\alpha'} f_3(\mp T) q_\pm^2 * 1 + q_\pm F^{(\pm)} \\
&\longrightarrow \quad -\frac{1}{4\pi\alpha'} q_\pm^2 f_3(\mp T) * 1\,.
\end{aligned}
$$

In the second line, we have introduced an auxiliary field $q_\pm$. The equation of motion for $q_\pm$ is

$$
q_\pm = -2\pi\alpha' f_3(\pm T) * F^{(\pm)}\,, \tag{4.3.1}
$$

which, when substituted in, gives the original action. In the third line, we have integrated out $A^{(\pm)}$ which constrains $q_\pm$ to be a constant. Therefore, in the third line, the fields $A^{(\pm)}$ and $q_\pm$ are no longer functionally integrated. The full action can now be written as

$$
\begin{aligned}
S = \int dx dt \sqrt{-g}\Bigg[\frac{e^{-2\Phi}}{2\kappa^2}\left(\frac{8}{\alpha'} + R + 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) + \dots\right) \\
- \frac{1}{4\pi\alpha'} f_3(-T) q_+^2 - \frac{1}{4\pi\alpha'} f_3(T) q_-^2 + \dots \Bigg]\,.
\end{aligned} \tag{4.3.2}
$$

Varying with respect to the metric $g_{\mu\nu}$, dilaton Φ, and tachyon T gives the equations of motion

$$(\delta\mathbf{g})\qquad \frac{1}{2}g^{\mu\nu}\left[\frac{e^{-2\Phi}}{2\kappa^2}\left(\frac{8}{\alpha'}+4\nabla^2\Phi-4(\nabla\Phi)^2-f_1(T)(\nabla T)^2+f_2(T)\right)\right.$$
$$\left.-\frac{1}{4\pi\alpha'}f_3(-T)q_+^2-\frac{1}{4\pi\alpha'}f_3(T)q_-^2\right]$$
$$+\frac{e^{-2\Phi}}{2\kappa^2}\left(-2\nabla^\mu\nabla^\nu\Phi+f_1(T)\nabla^\mu T\nabla^\nu T\right)=0\,, \quad (4.3.3)$$

$$(\delta\mathbf{\Phi})\qquad \frac{8}{\alpha'}+R+4\nabla^2\Phi-4(\nabla\Phi)^2-f_1(T)(\nabla T)^2+f_2(T)=0\,, \quad (4.3.4)$$

and

$$(\delta\mathbf{T})\qquad \frac{e^{-2\Phi}}{2\kappa^2}\left[2f_1(T)\nabla^2T+f_1'(T)(\nabla T)^2-4f_1(T)(\nabla_\mu\Phi)(\nabla^\mu T)+f_2'(T)\right]$$
$$-\frac{1}{4\pi\alpha'}f_3'(-T)q_+^2-\frac{1}{4\pi\alpha'}f_3'(T)q_-^2=0\,, \quad (4.3.5)$$

where primes denote differentiation with respect to T. Setting Φ and T constant, we find

$$(\delta\mathbf{g})\qquad \frac{e^{-2\Phi}}{2\kappa^2}\left(\frac{8}{\alpha'}+f_2(T)\right)-\frac{1}{4\pi\alpha'}q_+^2f_3(-T)-\frac{1}{4\pi\alpha'}q_-^2f_3(T)=0\,, \quad (4.3.6)$$

$$(\delta\mathbf{\Phi})\qquad \frac{8}{\alpha'}+R+f_2(T)=0\,, \quad (4.3.7)$$

and

$$(\delta\mathbf{T})\qquad \frac{e^{-2\Phi}}{2\kappa^2}f_2'(T)-\frac{1}{4\pi\alpha'}q_+^2f_3'(-T)-\frac{1}{4\pi\alpha'}q_-^2f_3'(T)=0\,. \quad (4.3.8)$$

With the AdS_2 metric

$$ds^2=\frac{-4l^2}{\sin^2(u^+-u^-)}du^+du^-\,, \quad (4.3.9)$$

the Ricci scalar is

$$R=-2/l^2\,. \quad (4.3.10)$$

4.3.2 Solutions $T = 0$

The solution with $q_- = q_+ \equiv q$ and $T = 0$ satisfies the equations of motion with AdS radius given by

$$l^2 = \alpha'/4 \tag{4.3.11}$$

and dilaton given by

$$e^{-2\Phi} = \frac{\kappa^2}{8\pi} q^2 \,. \tag{4.3.12}$$

A notable feature of this solution is that the curvature radius is fixed at a value of order the string length. This implies that higher order α' terms in the effective action will be important. This will be addressed in section 4.4. Also, note that we are free to tune the string coupling to zero by ramping up the strength of the R-R flux.

In this case, the "tachyon" is massive for all values of q. This can be seen as follows. The δT equation of motion, to first order in T, gives us

$$\left\{ \nabla^2 + \nabla^2\Phi - (\nabla\Phi)^2 + \frac{2}{\alpha'} - \frac{4\kappa^2}{\pi\alpha'} e^{2\Phi} q^2 \right\} \left(e^{-\Phi} T \right) = 0 \,. \tag{4.3.13}$$

The $\delta\Phi$ equation of motion, to zero order in T, tells us that

$$\nabla^2\Phi - (\nabla\Phi)^2 + \frac{2}{\alpha'} = -\frac{R}{4} \,, \tag{4.3.14}$$

and, when substituted into the linearized δT equation, gives us

$$\left\{ \nabla^2 - \frac{R}{4} - \frac{4\kappa^2}{\pi\alpha'} e^{2\Phi} q^2 \right\} \left(e^{-\Phi} T \right) = 0 \,. \tag{4.3.15}$$

Finally, substituting our background expressions for Φ and R, we get

$$\left(\nabla^2 - \frac{30}{\alpha'} \right) \left(e^{-\Phi} T \right) = 0 \,, \tag{4.3.16}$$

so that the tachyon mass is $m_T^2 = \frac{30}{\alpha'} = \frac{15}{2l^2}$. The authors of [26] noted that, in ten dimensions, R-R flux could stabilize the tachyon potential. In our two-dimensional case, we see that the R-R flux makes the otherwise-massless tachyon massive.

Solutions to the wave equation in AdS$_2$ are most readily attained in Poincare coordinates, in which

$$ds^2 = l^2 \frac{-dt^2 + dy^2}{y^2} \,. \tag{4.3.17}$$

In these coordinates, the wave equation takes the form

$$\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} - \frac{l^2 m_T^2}{y^2} \right) T(t, y) = 0 \,. \tag{4.3.18}$$

Using separation of variables, we can write the general time-dependent, positive-frequency solution as $e^{-i\omega t}\chi(y)$. The normalizable solution is readily obtained in terms of a Bessel function as

$$T_w(t, y) = e^{-i\omega t} \sqrt{\frac{y}{2}} J_{h_\pm - 1/2}(\omega y) \,, \tag{4.3.19}$$

where $h_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + 4l^2 m_T^2}$. The static solutions are obtained by noting that the wave equation

$$y^2 \frac{\partial^2}{\partial y^2} T = l^2 m_T^2 T \tag{4.3.20}$$

implies that $T \sim y^n$ where $n(n-1) = l^2 m_T^2$. Therefore, the general static solution is

$$T = a y^{h_+} + b y^{h_-} \,. \tag{4.3.21}$$

Note that, although these static solutions are non-normalizable, they may make an appearance as approximate solutions in regions of spacetime that are AdS-like.

Solutions to the wave equation in global coordinates are a little more difficult to come by, but they have been worked out in [39]. In the global coordinates

$$ds^2 = l^2 \frac{-d\tau^2 + d\sigma^2}{cos^2\sigma}\,, \tag{4.3.22}$$

the normalized positive-frequency solutions are

$$T_n(\tau,\sigma) = \Gamma(h) 2^{h-1} \sqrt{\frac{n!}{\pi\Gamma(n+2h)}}\, e^{-i(n+h)\tau} (\cos\sigma)^h C_n^h(\sin\sigma)\,, \tag{4.3.23}$$

where $n = 0, 1, 2, \ldots$, C_n^h is the Gegenbauer polynomial, and h is once again related to m_T^2 by $h(h-1) = l^2 m_T^2$. Note that, unlike in Poincare coordinates, the spectrum in global coordinates is discrete.

4.3.3 Solutions with $T \neq 0$

The solution given in the previous section can be deformed by moving the constant value of T away from zero. The solution is given by

$$l^2 = \frac{\alpha'/4}{1 + \frac{\alpha'}{8} f_2(T)}\,, \tag{4.3.24}$$

$$e^{-2\Phi} = \frac{\frac{\kappa^2}{16\pi}\left(q_+^2 f_3(-T) + q_-^2 f_3(T)\right)}{1 + \frac{\alpha'}{8} f_2(T)}\,, \tag{4.3.25}$$

and

$$\frac{q_-^2}{q_+^2} = \frac{f_3(-T)}{f_3(T)} \frac{8/\alpha' + f_2(T) - f_2'(T)/2}{8/\alpha' + f_2(T) + f_2'(T)/2}\,. \tag{4.3.26}$$

Again, it is clear that, for all solutions in this family, we can send the string coupling to zero while holding fixed both the tachyon vev T and the AdS radius l. This is accomplished by sending q_-^2 and q_+^2 to infinity while holding the ratio q_-^2/q_+^2 fixed.

It is not evident from these equations whether or not there exists an AdS_2 solution with one of the q's, say q_-, set to zero. Setting $q_- = 0$ would require a T of order 1, but to understand such large values of T would require a more complete knowledge of f_2. Specifically, $q_- = 0$ would require that

$$\frac{8}{\alpha'} + f_2(T) - \frac{1}{2} f_2'(T) = 0\,, \tag{4.3.27}$$

and it is not known whether this equation has solutions.

4.4 Discussion

It should be noted that the AdS spaces presented here are solutions to the first few terms in the effective action. Since the AdS radius is of order the string length, we expect higher order terms in α' to change some of the quantitative features of the solutions, such as the exact value of the AdS radius or the true mass of the tachyon. However, as we shall discuss here, the qualitative features of the AdS solutions are rather generic and are not expected to be changed by the higher order α' terms.

We can ask what other terms might make contributions to the equations of motion, and, therefore, might change features of the AdS solution. For simplicity, let us concentrate on the $T = 0$ solution found in section 4.3.2. Since we seek an AdS solution in which R, T and Φ are constant and $F^{\pm}$ is nondynamical, the most general term of interest in the dualized action is $\alpha'^{n-1} e^{(2m-2)\Phi} T^p R^n q_{\pm}^{2m}$. The dilaton dependence is fixed by the number of R-R field strengths in the monomial [40, 41]. Since we are considering the $T = 0$ solutions in this discussion, terms with $p > 1$ will not contribute to any of the field equations. The $p = 1$ terms will contribute to the δT variation, but the proposed symmetry [35] of the theory under $T \to -T$ and $q_+ \leftrightarrow q_-$ guarantees that the contribution to the equation of motion will be proportional to $(q_+^{2m} - q_-^{2m})$. Setting $q_+^2 = q_-^2 \equiv q^2$ as before, this term disappears from the field equations.

Having dispensed with terms involving T, we are left to focus on terms of the form $\alpha'^{n-1} e^{(2m-2)\Phi} R^n q^{2m}$. Under variation of the metric, the higher-order terms in the action

$$\sum_{n,m} c_{n,m} \int \sqrt{-g} e^{(2m-2)\Phi} \alpha'^{n-1} R^n q^{2m}$$

modify the δg field equation to

$$\frac{e^{-2\Phi}}{2\kappa^2}\frac{8}{\alpha'} - \frac{q^2}{2\pi\alpha'} + \sum_{n,m} c_{n,m}(1-n) e^{(2m-2)\Phi} \alpha'^{n-1} R^n q^{2m} = 0\,.$$

We still seek a one-parameter family of solutions (with $T = 0$) in which q^2 is proportional to $e^{-2\Phi}$, and we will denote the constant of proportionality as B:

$$q^2 = B e^{-2\Phi}\,.$$

The δg EOM may now be written as

$$8 - \frac{\kappa^2}{\pi} B - 2\kappa^2 \sum c_{n,m}(n-1)(\alpha' R)^n B^m = 0\,.$$

Similarly, the $\delta\Phi$ EOM becomes

$$8 + \alpha' R - 2\kappa^2 \sum c_{n,m}(m-1)(\alpha' R)^n B^m = 0\,.$$

So long as there are simultaneous solutions to these two equations for some negative R and positive B, a one-parameter family of AdS solutions exists in which the string coupling may be tuned towards zero. This one-parameter family of AdS solutions, with Ricci scalar R, would be parametrized by q^2 with $e^{2\Phi} = B/q^2$.

The evidence gathered here suggests that the qualitative structure of the AdS solutions is rather generic and is likely to be unaffected by terms higher order in α'. This fact motivates a search for the corresponding worldsheet sigma model describing type 0A strings propagating in these AdS_2 spaces. Because of the existence of nonzero R-R fluxes, the correct sigma model will most likely not be found using the NSR formalism. Fortunately, several other worldsheet formalisms have been developed that have allowed for quantization of the string in R-R backgrounds. For example, the hybrid formalism has been used to study superstring quantization in $AdS_3 \times S^3$ [42], $AdS_2 \times S^2$ backgrounds [43], and curved 2D backgrounds [44].

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Chapter 5

Supersymmetric Branes in $AdS_2 \times S^2 \times CY_3$

5.1 Introduction

$AdS_2 \times S^2 \times CY_3$ flux compactifications of string theory arise as the near-horizon geometries of type IIA black holes. The fluxes are determined from the black hole charges. The vector moduli of the Calabi-Yau threefold and the radius of the $AdS_2 \times S^2$ are also determined in terms of these charges via the attractor equations [45, 46]. These compactifications are interesting for several reasons. A central unsolved problem in string theory is to find - assuming it exists - a holographically dual CFT_1 for these compactifications.[1] Moreover recently a simple and unexpected connection was found between the partition function of the black hole and the topological string on the corresponding attractor Calabi-Yau [48]. In this chapter we will further our understanding of these compactifications by analyzing the problem of supersymmetric brane configurations.

Following some review in section 5.2, in section 5.3 the problem of supersymmetric branes is analyzed from the viewpoint of the four dimensional effective $\mathcal{N} = 2$

[1] For some cases a dual CFT_2 is known [47].

theory on $AdS_2 \times S^2$. This analysis is facilitated by the recent construction [49] of the κ-symmetric superparticle action carrying general electric and magnetic charges (u^I, v_I) in such theories. It is found that there is always a supersymmetric trajectory whose position is determined by the phase of the central charge $Z(u^I, v_I)$. In global AdS_2 coordinates

$$ds^2 = R^2(-\cosh^2\chi d\tau^2 + d\chi^2 + d\theta^2 + \sin^2\theta d\phi^2) \tag{5.1.1}$$

the supersymmetric trajectory is at

$$\tanh\chi = \frac{\mathrm{Re}Z}{|Z|}\,. \tag{5.1.2}$$

For the general case $\chi \neq 0$ this trajectory is accelerated by the electromagnetic forces. We further consider n-particle configurations with differing charges and differing central charges Z_i, $\quad i = 1, ..n$, constrained only by the condition that they all have the same sign for $\mathrm{Re}Z_i$. Surprisingly if the positions of the charges are each determined by 5.1.2, a common supersymmetry is preserved for the entire multiparticle configuration. This is quite different than the case of fluxless Calabi-Yau-Minkowski compactifications, where there is a common supersymmetry only if the charges are aligned. Supersymmetry preservation is possible only because of the enhanced near-horizon superconformal group. This phenomena should have a counterpart in higher AdS spaces and may be of interest for braneworld scenarios.

In section 5.4 we consider the problem from the ten-dimensional perspective. For simplicity we consider only the $AdS_2 \times S^2 \times CY_3$ geometries arising from $D0-D4$ Calabi-Yau black holes. Adapting the analysis of [50] to this context, we allow the wrapped branes to induce lower brane charges by turning on worldvolume field strengths. We will find that there are no static, supersymmetric D0-branes in global coordinates because they want to accelerate off to the boundary of AdS_2 (there are static BPS configurations in Poincaré coordinates). For a D2-brane embedded holomorphically in the Calabi-Yau, we will find that it is half BPS and sits at $\chi = \tanh^{-1}(\sin\beta_{CY})$. Here, β_{CY} is related to the amount of magnetic flux on the worldvolume. All D2-brane that are static with respect to a common global time in AdS_2 preserve the same set of half of the supersymmetries regardless of β_{CY}. Similar conclusions hold for D4, D6-branes wrapped on the Calabi-Yau. We also consider a D2-brane wrapped on the S^2 of the $AdS_2 \times S^2$ product and find that it is once again half BPS and sits at $\chi = \tanh^{-1}(\sin\beta_{S^2})$.

5.2 Preliminaries

In this section we briefly review some material which will be needed for our analysis. We are interested in type IIA string theory compactified on a Calabi-Yau 3-fold M, with 2-cycles labeled by α^A, where $A = 1, 2, \cdots, n \equiv h_{11}$. The low energy effective theory is $\mathcal{N} = 2$ supergravity coupled to n vector multiplets (and also $h_{21}+1$ hypermultiplets which are not relevant in our discussion). This theory can be described using special geometry [51–55] and here we will follow the notation of [51]. The scalar components of the vector multiplets are described in terms of projective coordinates X^I, $I = 0, 1, \cdots, n$. The prepotential $F(X^I)$ is holomorphic and homogeneous of degree 2 in the X^I's. In the large volume limit F is of the form

$$F = D_{ABC}\frac{X^A X^B X^C}{X^0} + \cdots \tag{5.2.1}$$

where $D_{ABC} = -\frac{1}{6}C_{ABC}$, C_{ABC} being the triple intersection number of the 4-cycles dual to α^A, which we denote by Σ_A.

Extremal black holes of magnetic and electric charge $(p^0 = 0, p^A, q_0, q_A)$ are realized as a D4-brane wrapped on 4-cycle $P = \sum p^A \Sigma_A$ bound with q_0 D0-branes, together with q_A gauge field fluxes through the 2-cycles α^A. The asymptotic values of the moduli fields $X^I, F_I \equiv \partial_I F$ at infinity can be arbitrary. However at the black hole horizon they approach the fixed point values determined from the "attractor equations" [45, 46]

$$p^I = \mathrm{Re}\, CX^I\,, \quad q_I = \mathrm{Re}\, CF_I\,. \tag{5.2.2}$$

Using the tree level prepotential 5.2.1, the fixed points of the moduli are [56, 57]

$$CX^0 = i\sqrt{\frac{D}{\hat{q}_0}}\,, \quad CX^A = p^A + \frac{i}{6}\sqrt{\frac{D}{\hat{q}_0}} D^{AB} q_B \tag{5.2.3}$$

where

$$D \equiv D_{ABC} p^A p^B p^C\,, \tag{5.2.4}$$

$$\hat{q}_0 \equiv q_0 + \frac{1}{12} D^{AB} q_A q_B\,, \tag{5.2.5}$$

$$D_{AB} \equiv D_{ABC} p^C\,, \tag{5.2.6}$$

$$D^{AB} D_{BC} = \delta^A_C\,. \tag{5.2.7}$$

The near horizon geometry of the 4D extremal black hole is $AdS_2 \times S^2$ with the moduli at their attractor values. We are interested in string theory on the global $AdS_2 \times S^2 \times M$ geometry. The radius R of AdS_2 and S^2, which is the same as the radius of the extremal black hole, is determined in terms of the charges (p^I, q_I) via

$$R = \sqrt{2}\,(D\hat{q}_0)^{\frac{1}{4}} \tag{5.2.8}$$

where hereafter we work mainly in four-dimensional Planck units.

The metric on the Poincaré patch of $AdS_2 \times S^2$ is

$$ds^2 = R^2\left(\frac{-dt^2 + d\sigma^2}{\sigma^2} + d\theta^2 + \sin^2\theta d\phi^2\right) \tag{5.2.9}$$

while the metric is

$$ds^2 = R^2(-\cosh^2 \chi d\tau^2 + d\chi^2 + d\theta^2 + \sin^2 \theta d\phi^2) \tag{5.2.10}$$

in global coordinates. In much of this chapter we deal with the case $q_A = 0$, and here the RR field strengths are

$$F_{(2)} = \frac{1}{R}\omega_{AdS_2}\,, \qquad F_{(4)} = \frac{1}{R}\omega_{S^2} \wedge J\,, \tag{5.2.11}$$

where $\omega_{AdS_2} = R^2 \cosh \chi d\tau \wedge d\chi$ is the volume form on AdS_2, $\omega_{S^2} = R^2 \sin \theta d\theta \wedge d\phi$ is the volume form on the S^2, and J is the Kähler form on the Calabi-Yau. In particular, the Kähler volume of the 2-cycles α^A are determined by the charges as

$$\frac{1}{2\pi\alpha'} \int_{\alpha^A} J = 2\pi p^A \sqrt{\frac{q_0}{D}}\,. \tag{5.2.12}$$

5.3 Four-dimensional analysis

Flux compactifications on a Calabi-Yau threefold are described by an effective $d = 4$, $\mathcal{N} = 2$ supergravity with an $AdS_2 \times S^2$ vacuum solution whose moduli are at the attractor point with charges (p^I, q_I). This theory contains zerobranes[2] with essentially arbitrary charges (u^I, v_I) arising from various wrapped brane configurations. The κ-symmetric worldline action of these zerobranes was determined in [49]. In this section we use the results of [49] to determine the possible supersymmetric worldline trajectories.

The Killing spinor equation is

$$\nabla_\mu \epsilon_A - \frac{i}{2}\epsilon_{AB} T^-_{\mu\nu}\gamma^\nu \epsilon^B = 0\,, \tag{5.3.1}$$

[2] We use the term zerobrane in a general sense and do not specifically refer here to a ten-dimensional D0-brane.

where ϵ^A, $\epsilon_A = (\epsilon^A)^*$ $(A = 1, 2)$ are chiral and anti-chiral R-symmetry doublets of spinors. T^- is the anti-self-dual part of the graviphoton field strength, satisfying

$$Z_{BH} = \frac{1}{4\pi}\int_{S^2} T^- = e^{-\mathcal{K}/2}\left(F_I p^I - X^I q_I\right) , \tag{5.3.2}$$

where $\mathcal{K} = -\ln i(\overline{X}^I F_I - X^I \overline{F}_I)$ is the Kähler potential. Define the phase of the central charge $e^{i\alpha} = Z_{BH}/|Z_{BH}|$. Then we can write $T^- = -ie^{i\alpha}(1 + i*)F$, where $F = \frac{1}{R}\omega_{AdS}$. In terms of the doublet of spinors (ϵ_1, ϵ^2) and (ϵ^1, ϵ_2), the Killing spinor equation can be written as

$$\nabla_\mu \epsilon + \frac{i}{2} e^{-i\alpha\gamma_5} \not{F} \gamma_\mu \sigma^2 \epsilon = 0\,. \tag{5.3.3}$$

Note that there is an ambiguity in choosing the overall phase of the moduli fields and the central charge,

$$X^I \to e^{i\theta} X^I\,, \quad F_I \to e^{i\theta} F_I\,, \quad \epsilon \to e^{\frac{i}{2}\theta\gamma_5}\epsilon\,, \tag{5.3.4}$$

so we are free to set $\alpha = 0$.

The solutions to the Killing spinor equation in global $AdS_2 \times S^2$ coordinates 5.2.10 are [58]

$$\epsilon = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\exp\left(\frac{i}{2}\tau\gamma^1\sigma^2\right) R(\theta,\phi)\epsilon_0 \tag{5.3.5}$$

$$R(\theta,\phi) \equiv \exp\left(-\frac{i}{2}(\theta - \pi/2)\gamma^{012}\sigma^2\right)\exp\left(-\frac{i}{2}\phi\gamma^{013}\sigma^2\right) \tag{5.3.6}$$

where ϵ_0 is a doublet of arbitrary constant spinors. Alternatively, in the Poincare metric 5.2.9, the Killing spinors are [59]

$$\epsilon = \sigma^{-1/2} R(\theta,\phi)\epsilon_0^+ \quad \text{and} \quad \epsilon = (\sigma^{1/2} + i\sigma^{-1/2} t\gamma^1\sigma^2) R(\theta,\phi)\epsilon_0^- \,, \tag{5.3.7}$$

where $\epsilon_0^\pm$ are constant spinors satisfying $-i\gamma^0\sigma^2\epsilon_0^\pm = \pm\epsilon_0^\pm$, and $R(\theta,\phi)$ denotes the rotation on the S^2 as in 5.3.5. Note that γ^μ are the *normalized* gamma matrices in the corresponding frame.

The zerobrane action constructed in [49] has a local κ-symmetry parameterized by a four-dimensional spinor doublet κ_A on the worldline. In addition the spacetime supersymmetries ϵ_A act non-linearly in Goldstone mode on the worldline fermions. In general [60], a brane configuration trajectory will preserve a spacetime supersymmetry generated by ϵ if the action on the worldvolume fermions can be compensated for by a κ transformation. This condition can typically be written

$$(1-\Gamma)\epsilon = 0 \tag{5.3.8}$$

where Γ is a matrix appearing in the κ-transformations. For the case at hand it follows from the results of [49] that the condition is

$$\begin{aligned} \epsilon_A + e^{i\varphi}\Gamma_{(0)}\epsilon_{AB}\epsilon^B &= 0 \qquad (5.3.9)\\ \epsilon^A + e^{-i\varphi}\Gamma_{(0)}\epsilon^{AB}\epsilon_B &= 0 \qquad (5.3.10)\end{aligned}$$

where $\Gamma_{(0)}$ is the gamma matrix projected to the zerobrane worldline, and $e^{i\varphi}$ is the phase of the central charge Z of the zerobrane,

$$Z = e^{-\mathcal{K}/2}\left(u^I F_I - v_I X^I\right) = e^{i\varphi}|Z|\,, \tag{5.3.11}$$

where (u^I, v_I) are its magnetic and electric charges. In terms of the spinor doublet, one can write 5.3.9 as

$$-ie^{-i\varphi\gamma_5}\Gamma_{(0)}\sigma^2\epsilon = \epsilon\,. \tag{5.3.12}$$

Let us solve the condition for 5.3.12 to hold along the world line of a zerobrane sitting at constant (χ, θ, ϕ). Writing the Killing spinor as

$$\epsilon = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\exp\left(\frac{i}{2}\tau\gamma^1\sigma^2\right)\epsilon_0' \tag{5.3.13}$$

where $\epsilon_0' = R(\theta, \phi)\epsilon_0$, it suffices to solve

$$-ie^{-i\varphi\gamma_5}\gamma^0\sigma^2\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\epsilon_0' = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\epsilon_0' \tag{5.3.14}$$

$$-ie^{-i\varphi\gamma_5}\gamma^0\sigma^2\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2\epsilon_0' = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2\epsilon_0'\,. \tag{5.3.15}$$

Some straightforward algebra simplifies the above equations to

$$-i\gamma^0\sigma^2\left(\cos\varphi + i\cosh\chi\sin\varphi\gamma_5 + \sinh\chi\sin\varphi\gamma_5\gamma^0\sigma^2\right)\epsilon_0' = \epsilon_0' \tag{5.3.16}$$

$$i\gamma^0\sigma^2\left(\cos\varphi - i\cosh\chi\sin\varphi\gamma_5 + \sinh\chi\sin\varphi\gamma_5\gamma^0\sigma^2\right)\epsilon_0' = \epsilon_0'\,. \tag{5.3.17}$$

A solution exists only when

$$\tanh\chi = \cos\varphi\,, \tag{5.3.18}$$

and therefore $\cosh\chi\sin\varphi = \pm 1$. Correspondingly, the constraints on ϵ_0' become

$$\gamma_5\gamma^0\sigma^2\epsilon_0' = \mp\epsilon_0'\,, \tag{5.3.19}$$

where the sign on the RHS depends on the sign of $\sin\varphi$. This may be written as a condition on ϵ_0,

$$\left(1 \pm e^{\frac{i}{2}\phi\gamma^{013}\sigma^2}e^{i(\theta-\pi/2)\gamma^{012}\sigma^2}e^{\frac{i}{2}\phi\gamma^{013}\sigma^2}\gamma_5\gamma^0\sigma^2\right)\epsilon_0 = 0\,, \tag{5.3.20}$$

which makes it clear that zerobranes sitting at antipodal points on the S^2 will preserve opposite halves of the spacetime supersymmetries.

We conclude that a zerobrane following its charged geodesic in $AdS_2 \times S^2$ is half

BPS. The "extremal" case $\varphi = 0$ and π corresponds to the probe zerobrane with its charge aligned or anti-aligned with the charge of the original black hole. They cannot be stationary with respect to global time in the AdS_2. Using the Killing spinors on the Poincaré patch 5.3.7, it is clear that the "extremal" zerobranes following their charged geodesics (static on the Poincaré patch) are also half BPS. In the special case $\varphi = \pi/2$ in 5.3.18 the zerobrane moves along an uncharged geodesic and experiences no electromagnetic forces . This corresponds to the case when the zerobrane charge is orthogonal to all the black hole charges.

A somewhat surprising feature is that there are supersymmetric *multiparticle* configurations of zerobranes with *unaligned* charges. All "positively-charged" zerobranes with $0 < \varphi < \pi$ preserve the same set of half of the supersymmetries, and all "negatively-charged" zerobranes with $-\pi < \varphi < 0$ preserve the other set. Using the attractor equations the positive charge condition can be written in terms of the symplectic product of the black hole and zerobrane charges as

$$u^I q_I - p^I v_I > 0\,. \tag{5.3.21}$$

Given an arbitrary collection of zerobranes obeying 5.3.21 there is a half BPS configuration with the position of each trajectory determined in terms of the charges of the zerobrane by 5.3.18. Of course, such a supersymmetric configuration of particles with unaligned charges is not possible in the full black hole geometry prior to taking the near horizon limit. The preserved supersymmetry is part of the enhanced near-horizon supergroup.

This result is consistent with the expectation from the BPS bound. The energy of a charged zerobrane sitting at position χ the AdS_2 is given by

$$H = |Z| \cosh\chi - \frac{\mathrm{Re}(Z\bar{Z}_{BH})}{|Z_{BH}|} \sinh\chi = |Z| \left(\cosh\chi - \cos\varphi \sinh\chi\right)\,. \tag{5.3.22}$$

where the first term comes from the gravitational warping, and the second term comes from the coupling to the gauge field potential. At the stationary point $\tanh\chi = \cos\varphi$, the energy of the zerobrane is

$$|Z\sin\varphi| = \frac{|\mathrm{Im} Z\bar{Z}_{BH}|}{|Z_{BH}|}\,. \tag{5.3.23}$$

Therefore, as long as $\mathrm{Im}(Z\bar{Z}_{BH})$ is always positive (or negative), the BPS energy for multiple zerobranes is additive, in agreement with the supersymmetry analysis above.

5.4 Ten-dimensional analysis

In this section we analyze supersymmetric brane configurations from the point of view of the ten-dimensional IIA theory on $AdS_2 \times S^2 \times CY_3$. For simplicity we will focus on specific examples rather than the most general solution.

The extremal black hole in type IIA string theory compactified on a Calabi-Yau manifold M preserves four supersymmetries. After we take the near horizon limit, the number of preserved supersymmetries doubles to eight. We consider a background with only D0 and D4-brane charges, i.e. $q_A = p^0 = 0$, so that according to the attractor equations there is no B-field. The RR field strengths in the resulting $AdS_2 \times S^2 \times M_6$ are given as in 5.2.11. As shown in Appendix A, the ten-dimensional Killing spinor doublet is of the form

$$\varepsilon_1 = \epsilon_1 \otimes \eta_+ + \epsilon^1 \otimes \eta_-\,, \tag{5.4.1}$$

$$\varepsilon_2 = \epsilon^2 \otimes \eta_+ + \epsilon_2 \otimes \eta_-\,, \tag{5.4.2}$$

where $\eta_+, \eta_- = \eta_+^*$ are the chiral and anti-chiral covariantly constant spinors on M; $\epsilon_A = (\epsilon^A)^*$, $\epsilon^{1,2}$ are four-dimensional chiral spinors satisfying the four-dimensional

Killing spinor equation

$$\nabla_\mu \epsilon_A + \frac{i}{2} \not{F}^{(2)} \gamma_\mu (\sigma^2)_{AB} \epsilon^B = 0 \,. \tag{5.4.3}$$

This is the same equation as 5.3.3 with $\alpha = 0$, and the solutions are given by 5.3.5, 5.3.7.

We want to find all the BPS configurations of D-branes that are wrapped on compact portions of our background, and are pointlike in the AdS_2. In order for the D-brane to be supersymmetric, we only need to check that the κ-symmetry constraint

$$\Gamma \varepsilon = \varepsilon \tag{5.4.4}$$

is satisfied, where ε is the Killing spinor corresponding to the unbroken supersymmetry (more precisely, the pullback onto the brane world volume). The κ projection matrix is given by [61–64]

$$\Gamma = \frac{\sqrt{\det G}}{\sqrt{\det(G + \mathcal{F})}} \sum_n \frac{1}{2^n n!} \Gamma^{\hat{\mu}_1 \hat{\nu}_1 \cdots \hat{\mu}_n \hat{\nu}_n} \mathcal{F}_{\hat{\mu}_1 \hat{\nu}_1} \cdots \mathcal{F}_{\hat{\mu}_n \hat{\nu}_n} \Gamma_{(10)}^{n + \frac{p-2}{2}} \Gamma_{(0)} \sigma^1 \,, \tag{5.4.5}$$

$$\Gamma_{(0)} = \frac{1}{(p+1)! \sqrt{\det G}} \epsilon^{\hat{\mu}_0 \cdots \hat{\mu}_p} \Gamma_{\hat{\mu}_0 \cdots \hat{\mu}_p} \,, \tag{5.4.6}$$

where the hatted indices label coordinates on the brane world-volume, G is the pullback of the spacetime metric, and $\mathcal{F} = F + f^*(B)$ (the B-field is zero in our discussion). See Appendix A for conventions on 10D gamma matrices. Unless otherwise noted we will work in global coordinates 5.2.10.

5.4.1 D0-brane

For a static D0-brane in global coordinates, we have $\Gamma_{(0)} = \gamma^0$. The κ-symmetry matrix is

$$\Gamma = \Gamma_{(10)} \gamma^0 \sigma^1 \tag{5.4.7}$$

Writing the doublet ε in terms of the 4-dimensional spinor doublet ϵ

$$\varepsilon = \epsilon \otimes \eta_+ + \epsilon^* \otimes \eta_- \,, \tag{5.4.8}$$

The matrix Γ acts on ε as $\gamma^0\sigma^1\sigma^3 = -i\gamma^0\sigma^2$. The κ-symmetry constraint 5.4.4 becomes

$$(1 + i\gamma^0\sigma^2)\epsilon = 0 \,. \tag{5.4.9}$$

Using the explicit solutions of the Killing spinors in global AdS 5.3.5, we see that 5.4.9 cannot be satisfied at all τ, so a D0-brane static in global AdS can never be BPS. This is of course expected since the charged geodesic cannot be static in global coordinates. On the other hand, using 5.3.7 we see that a D0-brane static with respect to the Poincaré time is always half BPS, as expected.

5.4.2 D2 wrapped on Calabi-Yau, $F = 0$

Now let us consider a D2-brane wrapped on M and static in global $AdS_2 \times S^2$, without any world-volume gauge fields turned on. The κ-symmetry matrix is

$$\Gamma = \frac{1}{2\sqrt{\det' G}} \gamma^0 \epsilon^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} \sigma^1 \tag{5.4.10}$$

where $\det'$ takes the determinant of the spatial components of the world volume metric. Acting on ε, we have

$$\begin{aligned}
\Gamma_{\hat{a}\hat{b}}\varepsilon &= \partial_{\hat{a}} X^I \partial_{\hat{b}} X^J \gamma_{IJ} \varepsilon && (5.4.11)\\
&= 2\partial_{\hat{a}} X^i \partial_{\hat{b}} X^{\bar{j}} \gamma_{i\bar{j}} \varepsilon + \partial_{\hat{a}} X^i \partial_{\hat{b}} X^j \gamma_{ij} \varepsilon + \partial_{\hat{a}} X^{\bar{i}} \partial_{\hat{b}} X^{\bar{j}} \gamma_{\bar{i}\bar{j}} \varepsilon && (5.4.12)\\
&= 2\partial_{\hat{a}} X^i \partial_{\hat{b}} X^{\bar{j}} \left(-g_{i\bar{j}} \gamma_{(6)}\right) \varepsilon + \left(\frac{1}{2} \partial_{\hat{a}} X^i \partial_{\hat{b}} X^j \Omega_{ijk} \epsilon \otimes \gamma^k \eta_- + c.c.\right). && (5.4.13)
\end{aligned}$$

The κ-symmetry constraint $\Gamma\varepsilon = \varepsilon$ implies $\epsilon^{\hat{a}\hat{b}}\partial_{\hat{a}}X^i\partial_{\hat{b}}X^j\Omega_{ijk} = 0$, which means that the D2-brane must wrap a holomorphic 2-cycle. It then follows that Γ acts on ε as $\Gamma\varepsilon = i\gamma^0\gamma_{(6)}\sigma^1\varepsilon = \gamma_{(4)}\gamma^0\sigma^2\varepsilon$. Therefore 5.4.4 becomes

$$(1 - \gamma_{(4)}\gamma^0\sigma^2)\epsilon = 0\,. \tag{5.4.14}$$

It is clear that the wrapped D2-brane sitting at $\chi = 0$ in AdS_2 is half BPS. Note that the D2-brane without gauge field flux doesn't feel any force due to the RR fluxes ($q_A = 0$), so its stationary position is at the center of AdS_2.

5.4.3 D2 wrapped on Calabi-Yau, $F \neq 0$

With general worldvolume gauge field strength F turned on, the matrix Γ is

$$\Gamma = \frac{1}{\sqrt{\det'(G+F)}}\left(1 + \frac{1}{2}\Gamma^{\hat{a}\hat{b}}F_{\hat{a}\hat{b}}\Gamma_{(10)}\right)\gamma^0\left(\frac{1}{2}\epsilon^{\hat{c}\hat{d}}\Gamma_{\hat{c}\hat{d}}\right)\sigma^1 \tag{5.4.15}$$

An argument nearly identical to the one given in [50] shows that the supersymmetric D2-brane must wrap a holomorphic 2-cycle, and the gauge flux F satisfies

$$\frac{\sqrt{\det G}}{\sqrt{\det(G+F)}}(f^*J + iF) = e^{i\beta}\mathrm{vol}_2 \tag{5.4.16}$$

where vol_2 is the volume form on the D2-brane (which is just f^*J for a holomorphically wrapped brane), and β is a constant phase determined in terms of the D0-brane charge $2\pi n = \frac{1}{2\pi\alpha'}\int F$ via

$$\frac{\tan\beta}{2\pi\alpha'}\int J = 2\pi n\,. \tag{5.4.17}$$

If the probe D2-brane is wrapped on the 2-cycle $[\Sigma_2] = n_A\alpha^A$, then using 5.2.12 we have

$$\tan\beta = \frac{n}{n_A p^A}\sqrt{\frac{D}{q_0}} \tag{5.4.18}$$

Note that from 5.4.16 we have $\cos\beta > 0$, since J is positive when restricted to holomorphic cycles. The κ-symmetry condition then becomes

$$(1 - e^{-i\beta\gamma_{(4)}}\gamma_{(4)}\gamma^0\sigma^2)\epsilon = 0 \tag{5.4.19}$$

These is identical to 5.3.12 if we set $\varphi = \beta - \pi/2$. We can immediately read off the conditions for the static D2-brane to preserve supersymmetry when it sits at $\theta = \pi/2$, $\phi = 0$ in the S^2:

$$\sin\beta = \tanh\chi\,, \qquad \cos\beta = \mathrm{sech}\chi\,, \qquad (1 - \gamma_{(4)}\gamma^0\sigma^2)\epsilon_0 = 0\,. \tag{5.4.20}$$

We see that for general $-\pi/2 < \beta < \pi/2$, the D2-brane sits at $\chi = \tanh^{-1}(\sin\beta)$ and is half BPS. In fact they all preserve the same half supersymmetries, as discuss in section 5.3. Anti-D2-branes with gauge field fluxes wrapped on holomorphic 2-cycles will preserve the other half supersymmetries.

5.4.4 Higher dimensional D-branes wrapped on the Calabi-Yau

Let us consider D4, D6-branes that are wrapped on the Calabi-Yau and sit at constant position in global $AdS_2 \times S^2$. We shall use a trick [64] to write the matrix Γ as

$$\Gamma = e^{-A/2}\Gamma_{(10)}^{\frac{p-2}{2}}\Gamma_{(0)}e^{A/2}\sigma^1 \tag{5.4.21}$$

where

$$A = -\frac{1}{2}Y_{\hat{a}\hat{b}}\Gamma^{\hat{a}\hat{b}}\Gamma_{(10)} \tag{5.4.22}$$

and $Y_{\hat{a}\hat{b}}$ is an anti-symmetric matrix (analogous to the phase β in the previous subsection), related to the gauge field strength matrix $F_{\hat{a}\hat{b}}$ by

$$F = \tanh Y \tag{5.4.23}$$

By the same arguments as before, one can show that the BPS D-branes must wrap holomorphic cycles. Note that A acts on the Killing spinor ε as $A\varepsilon = -iY_{\hat{a}\hat{b}}(f^*J)^{\hat{a}\hat{b}}\gamma_{(4)}\varepsilon$, and $\Gamma_{(0)}$ acts as $\gamma^0(i\gamma_{(6)})^{p/2}$ (see Appendix). Let us define $\beta = -Y_{\hat{a}\hat{b}}(f^*J)^{\hat{a}\hat{b}}$. The κ-symmetry constraint can be written as

$$\Gamma\varepsilon = e^{-i\beta\gamma_{(4)}/2}\Gamma_{(10)}^{\frac{p-2}{2}}\gamma^0(i\gamma_{(6)})^{p/2}e^{i\beta\gamma_{(4)}/2}\sigma^1\varepsilon = \varepsilon\,. \tag{5.4.24}$$

We can simplify this to

$$-ie^{-i(\beta-p\pi/4)\gamma_{(4)}}\gamma^0\sigma^2\epsilon = \epsilon\,. \tag{5.4.25}$$

This equation indeed agrees with 5.4.9, 5.4.19 in the cases $p = 0, 2$. It is also identical to 5.3.12 provided we set $\varphi = \beta - p\pi/4$. So we conclude that a general Dp-brane (p even) wrapped on a holomorphic cycle in the Calabi-Yau, possibly with world-volume gauge fields turned on, static in the S^2 and following its charged geodesic in the AdS_2 is half BPS. As in [50] there is a deformation of the supersymmetry condition on the worldvolume gauge field F. In particular, the D-brane sits at $\tanh\chi = \cos(\beta - p\pi/4)$.

5.4.5 D2 wrapped on S^2, $F = 0$

Now let us turn to D2-branes wrapped on the S^2 appearing in the the $AdS_2 \times S^2 \times M$ product. The κ-symmetry matrix is $\Gamma = \Gamma_{(0)}\sigma^1 = \gamma^{023}\sigma^1$. 5.4.4 can be written as

$$(1 - \gamma^{023}\sigma^1)\epsilon = 0\,. \tag{5.4.26}$$

Defining $R(\theta,\phi)$ to be the S^2-dependent factors in 5.3.5, this condition becomes

$$(1-\gamma^{023}\sigma^1)\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta,\phi)\epsilon_0 = 0\,, \tag{5.4.27}$$

$$(1-\gamma^{023}\sigma^1)\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2 R(\theta,\phi)\epsilon_0 = 0\,. \tag{5.4.28}$$

A little algebra reduces these to

$$\cosh\frac{\chi}{2}(1-\gamma^{023}\sigma^1)R(\theta,\phi)\epsilon_0 = \sinh\frac{\chi}{2}(1+\gamma^{023}\sigma^1)R(\theta,\phi)\epsilon_0 = 0\,. \tag{5.4.29}$$

The only way to satisfy both equations is to set $\chi = 0$. Since $\gamma^{023}\sigma^1$ commutes with $R(\theta,\phi)$, we end up with the condition

$$(1-\gamma^{023}\sigma^1)\epsilon_0 = 0\,. \tag{5.4.30}$$

We conclude that the D2-brane sitting at the center of AdS and wrapped on the S^2 is half BPS.

5.4.6 D2 wrapped on S^2, $F \neq 0$

With gauge field strength $F = f\omega_{S^2}$ turned on, the κ-symmetry matrix acts on ε as

$$\begin{aligned}
\Gamma\varepsilon &= \frac{\sqrt{\det G}}{\sqrt{\det(G+F)}}\left(1+\frac{1}{2}\Gamma^{\hat{a}\hat{b}}F_{\hat{a}\hat{b}}\Gamma_{(10)}\right)\Gamma_{(0)}\sigma^1\varepsilon & (5.4.31)\\
&= \frac{1}{\sqrt{1+f^2}}\left(1+\gamma^{23}f\Gamma_{(10)}\right)\gamma^{023}\sigma^1\varepsilon & (5.4.32)\\
&= \exp\left(\beta\gamma^{23}\Gamma_{(10)}\right)\gamma^{023}\sigma^1\varepsilon = \gamma^{023}\sigma^1\exp\left(\beta\gamma^{23}\sigma^3\right)\varepsilon\,, & (5.4.33)
\end{aligned}$$

where $f \equiv \tan\beta$ ($\cos\beta > 0$). The condition 5.4.4 then becomes

$$(1 - \cos\beta\gamma^{023}\sigma^1 - i\sin\beta\gamma^0\sigma^2)\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta,\phi)\epsilon_0 = 0\,, \tag{5.4.34}$$

$$(1 - \cos\beta\gamma^{023}\sigma^1 + i\sin\beta\gamma^0\sigma^2)\exp\left(\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta,\phi)\epsilon_0 = 0\,, \tag{5.4.35}$$

A little algebra yields

$$(1 + \sin\beta\coth\chi)\,\epsilon_0 = 0\,, \tag{5.4.36}$$

$$\left(1 + \gamma^{023}\sigma^1\cot\beta\sinh\chi\right)\epsilon_0 = 0\,. \tag{5.4.37}$$

This means that $\sin\beta = -\tanh\chi$. In particular β, hence f, is constant on the world-volume. The condition on ϵ_0 becomes

$$(1 - \gamma^{023}\sigma^1)\epsilon_0 = 0\,. \tag{5.4.38}$$

These D-brane configurations are again half BPS.

5.4.7 D-branes wrapped on S^2 and the Calabi-Yau

In general for a Dp-branes wrapped on S^2 times some $(p-2)$-cycle in the Calabi-Yau, and static in global AdS_2, the matrix Γ is essentially the product of the piece on S^2 and the piece on Calabi-Yau,

$$\Gamma\varepsilon = \exp\left(-\beta_{S^2}\gamma^{23}\sigma^3\right)\exp\left(-i\beta_{CY}\gamma_{(4)}\right)\left(i\gamma_{(4)}\right)^{\frac{p-2}{2}}\gamma^{023}\sigma^1\varepsilon \tag{5.4.39}$$

where β_{CY} and β_{S^2} are the phases related to the world-volume gauge flux along the Calabi-Yau and S^2 directions as before. Define $\varphi_{CY} = \beta_{CY} - (p-2)\pi/4$, $\varphi_{S^2} =$

$\beta_{S^2} + \pi/2$. The κ-symmetry constraint can be written as

$$-i \exp\left(-\varphi_{S^2}\gamma^{23}\sigma^3 - i\varphi_{CY}\gamma_{(4)}\right)\gamma^0\sigma^2\epsilon = \epsilon \tag{5.4.40}$$

This is equivalent to

$$\left[1 + i\exp\left(-\varphi_{S^2}\gamma^{23}\sigma^3 - i\varphi_{CY}\gamma_{(4)}\right)\gamma^0\sigma^2\right]\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta,\phi)\epsilon_0 = 0, \tag{5.4.41}$$

$$\left[1 - i\exp\left(\varphi_{S^2}\gamma^{23}\sigma^3 + i\varphi_{CY}\gamma_{(4)}\right)\gamma^0\sigma^2\right]\exp\left(\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta,\phi)\epsilon_0 = 0. \tag{5.4.42}$$

A little algebra yields

$$\left[\sinh\chi - \cosh\chi\cos(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY})\right]R(\theta,\phi)\epsilon_0 = 0\,, \tag{5.4.43}$$

$$\left[\cosh\chi - \sinh\chi\cos(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY})\right. \tag{5.4.44}$$

$$\left. -\gamma^{023}\sigma^1\sin(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY})\right]R(\theta,\phi)\epsilon_0 = 0\,, \tag{5.4.45}$$

If φ_{CY} and φ_{S^2} are both nonzero, the first equation can be satisfied only if

$$i\gamma_{(4)}\gamma^{23}\sigma^3 R(\theta,\phi)\epsilon_0 = mR(\theta,\phi)\epsilon_0\,, \qquad m = \pm 1\,. \tag{5.4.46}$$

However, since $\gamma_{(4)}\gamma^{23}\sigma^3$ does not commute with $R(\theta,\phi)$ at generic points on the S^2, 5.4.46 can never be satisfied. Therefore such wrapped D-branes cannot be BPS.

If $\varphi_{S^2} = 0$, $\varphi_{CY} \neq 0$, we have

$$\tanh\chi = \cos\varphi_{CY} \tag{5.4.47}$$

and

$$(1 - \gamma_{(4)}\gamma^0\sigma^2)R(\theta,\phi)\epsilon_0 = 0\,. \tag{5.4.48}$$

However, in this case again $\gamma_{(4)}\gamma^0\sigma^2$ does not commute with $R(\theta,\phi)$ for generic (θ,ϕ), and hence 5.4.48 has no solution.

If $\varphi_{S^2} \neq 0$, $\varphi_{CY} = 0$, we find

$$\tanh \chi = \cos \varphi_{S^2} \tag{5.4.49}$$

and the second equation in 5.4.43 becomes

$$(1 - \gamma^{023}\sigma^1)\epsilon_0 = 0 \tag{5.4.50}$$

We see that such D-branes are half BPS.

So far we have neglected an important subtlety. For D4 or D6-branes wrapped on S^2 times some cycle in the Calabi-Yau, the RR flux $F_{(4)}$ induces couplings of gauge fields on the brane world-volume

$$\int_{D4} A \wedge F_{(4)}\,, \tag{5.4.51}$$
$$\int_{D6} A \wedge F \wedge F_{(4)}\,, \tag{5.4.52}$$

Since $F_{(4)} = \frac{1}{R}\omega_{S^2}\wedge J$, we see that for the D4-brane wrapped on $S^2\times\Sigma_2$ ($[\Sigma_2] = n_A\alpha^A$), the RR flux induces an electric charge density on the brane world-volume, of total charge

$$Q = \frac{1}{2\pi g_s}\int_{S^2\times\Sigma_2} F_{(4)} = \sum n_A p^A \tag{5.4.53}$$

Since the world-volume is compact, the Gauss law constraint requires the total charge to vanish. So we cannot wrap only a single D4-brane on $S^2 \times \Sigma$. One must introduce fundamental strings ending on the brane to cancel the electric charges. We then have $\sum n_A p^A$ fundamental strings ending on the D4-brane, and runoff to the boundary of AdS. This is interpreted as a classical "baryon" in the dual CFT.

Similarly for the D6-brane wrapped on $S^2 \times \Sigma_4$, one would have nonzero total electric charge on the world-volume if $\int_{\Sigma_4} F \wedge J \neq 0$. This again corresponds to certain "baryons" in the dual CFT.

Finally, a D6-brane wrapped on $S^2 \times \Sigma_4$ with general gauge field flux in the S^2 is half BPS, as shown in 5.4.49, 5.4.50.

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5.5 Appendix. The 10-dimensional Killing spinors

In order to write a ten-dimensional spinor as the tensor product of four-dimensional and internal (Calabi-Yau) spinors, it is necessary to work with a tensor product of Clifford algebras. Let Γ^M denote the ten-dimensional Clifford algebra matrices, with $M = 0, \ldots, 10$, $\mu = 0, \ldots, 3$, and $m = 4, \ldots, 9$. We can decompose the Γ^M into a tensor product of four and six-dimensional Clifford matrices, denoted by γ^μ and γ^m, as

$$\Gamma^\mu = \gamma^\mu \otimes 1, \tag{5.5.1}$$

$$\Gamma^m = \gamma_{(4)} \otimes \gamma^m \,. \tag{5.5.2}$$

Using a mostly-positive metric signature, the following matrices have the desired properties that they anticommute with the appropriate gamma matrices and square to one:

$$\Gamma_{(10)} = -\Gamma^{0123456789}, \tag{5.5.3}$$

$$\gamma_{(4)} = i\gamma^{0123}, \tag{5.5.4}$$

$$\gamma_{(6)} = i\gamma^{456789}\,. \tag{5.5.5}$$

With these sign conventions, $\Gamma_{(10)}$ decomposes in the desired way as $\Gamma_{(10)} = \gamma_{(4)} \otimes \gamma_{(6)}$.

As an ansatz for the Killing spinors, we assume they take the form

$$\varepsilon_1 = \epsilon_1 \otimes \eta_+ + \epsilon^1 \otimes \eta_-\,, \qquad \varepsilon_2 = \epsilon^2 \otimes \eta_+ + \epsilon_2 \otimes \eta_-\,, \tag{5.5.6}$$

where the ε's are 10D Majorana-Weyl spinors, the η's are 6D covariantly-constant Weyl spinors on the Calabi-Yau, and the ϵ's are 4D Majorana spinors. We use chiral notation in which the chirality of the spinor is denoted by the position of the R-symmetry index. In particular, $\epsilon(A) = \epsilon^A + \epsilon_A$ where $\gamma_{(4)}\epsilon^A = \epsilon^A$ and $\gamma_{(4)}\epsilon_A = -\epsilon_A$. Of course, there are no Majorana-Weyl spinors in $3+1$ dimensions; the four-dimensional chiral projections are related by $\epsilon_A = \epsilon^{A*}$. For the six-dimensional Weyl spinors, we use the standard notation where $\gamma_{(6)}\eta_\pm = \pm\eta_\pm$. Since we will work with type IIA, the tensor products have been chosen such that the ten-dimensional spinors are of opposite chirality. In doublet notation,

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \tag{5.5.7}$$

$\Gamma_{(10)}\varepsilon$ can be written as $-\sigma^3\varepsilon$. In addition, the following identities for the spinors $\eta_\pm$

will be useful:

$$\gamma_{\bar{i}}\eta_+ = 0\,, \quad \gamma_{ijk}\eta_+ = \Omega_{ijk}\eta_-\,, \quad \gamma_{ij}\eta_+ = \frac{1}{2}\Omega_{ijk}\gamma^k\eta_-\,, \quad \gamma_{\bar{i}\bar{j}kl}\eta_+ = \left(g_{k\bar{j}}g_{l\bar{i}} - g_{k\bar{i}}g_{l\bar{j}}\right)\eta_+\,, \tag{5.5.8}$$

$$\gamma_i\eta_- = 0\,, \quad \gamma_{\bar{i}\bar{j}\bar{k}}\eta_- = \Omega_{\bar{i}\bar{j}\bar{k}}\eta_+\,, \quad \gamma_{\bar{i}\bar{j}}\eta_- = \frac{1}{2}\Omega_{\bar{i}\bar{j}\bar{k}}\gamma^{\bar{k}}\eta_+\,, \quad \gamma_{ij\bar{k}\bar{l}}\eta_- = \left(g_{\bar{k}j}g_{\bar{l}i} - g_{\bar{k}i}g_{\bar{l}j}\right)\eta_-\,. \tag{5.5.9}$$

Given these ansätze, we want to check that the supersymmetry variations of the background vanish modulo conditions on the four-dimensional Majorana components of the Killing spinors. Since we work only with bosonic backgrounds, we need only check the variations of dilatino and gravitino.

The supersymmetry variation of the dilatino is [65]

$$\delta\lambda = \frac{1}{2}\left(3\not{F}_{(2)}i\sigma^2 + \not{F}_{(4)}\sigma^1\right)\varepsilon\,, \tag{5.5.10}$$

where $F_{(2)} = \frac{1}{R}\omega_{AdS_2}$ and $F_{(4)} = \frac{1}{R}\omega_{S^2}\wedge J$. Taking note of the fact that $g^{\bar{i}j}\gamma_{\bar{i}j}\eta_\pm = 3\gamma_{(6)}\eta_\pm$ and $\not{\omega}_{S^2} = -i\not{\omega}_{AdS_2}\gamma_{(4)}$, we find that

$$\not{F}_{(4)}\varepsilon = -3i\not{\omega}_{AdS_2}\gamma_{(4)}\gamma_{(6)}\varepsilon = -3\not{F}_{(2)}\sigma^3\varepsilon\,. \tag{5.5.11}$$

As a result, the dilatino variation vanishes automatically.

The gravitino variation is

$$\delta\psi_M = \nabla_M\varepsilon + \frac{1}{8}\left(\not{F}_{(2)}\Gamma_M i\sigma^2 + \not{F}_{(4)}\Gamma_M\sigma^1\right)\varepsilon = 0\,. \tag{5.5.12}$$

When the free index is holomorphic in the Calabi-Yau, this reduces to the following condition:

$$\left(\not{F}_{(2)}\gamma_m i\sigma^2 + \not{F}_{(4)}\gamma_m\sigma^1\right)\varepsilon = 0\,. \tag{5.5.13}$$

Using the fact that $g^{i\bar{j}}\gamma_{i\bar{j}}\gamma_m\eta_\pm = \gamma_m\gamma_{(6)}\eta_\pm$, we find that $\not{F}_{(4)}\gamma_m\varepsilon = -\not{F}_{(2)}\gamma_m\sigma^3\varepsilon$. This works similarly for an antiholomorphic index, so the gravitino variation is identically zero when the free index is in the Calabi-Yau.

When the gravitino equation has its free index in the $AdS_2 \times S^2$ space, the variation becomes

$$\delta\psi_\mu = \left[\nabla_\mu \pm \frac{1}{8}\gamma_\mu\left(\not{F}_{(2)}i\sigma^2 - \sigma^1\not{F}_{(4)}\right)\right]\varepsilon = 0\,, \tag{5.5.14}$$

where the $\pm$ is $+$ if μ is in the S^2 and $-$ if μ is in the AdS_2. Using the same identity used for the dilatino equation, we get

$$\delta\psi_\mu = \left[\nabla_\mu \pm \frac{i}{2}\gamma_\mu\,\not{F}_{(2)}\sigma^2\right]\varepsilon = \left[\nabla_\mu + \frac{i}{2}\,\not{F}_{(2)}\gamma_\mu\sigma^2\right]\varepsilon\,. \tag{5.5.15}$$

Demanding that the terms linear in η_+ and linear in η_- must vanish separately, we get the 4D equations

$$\left[\nabla_\mu + \frac{i}{2}\not{F}_{(2)}\gamma_\mu\sigma^2\right]\epsilon = 0\,, \tag{5.5.16}$$

where $\epsilon = \begin{pmatrix}\epsilon_1\\ \epsilon^2\end{pmatrix}$.

It is useful to derive the action of $\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{\det G}}\epsilon^{\hat{\mu}_0\cdots\hat{\mu}_p}\Gamma_{\hat{\mu}_0\cdots\hat{\mu}_p}$ on the $\eta_\pm$ which live on the world-volume of holomorphically wrapped D-branes (see 5.4.5). For D0-branes we have simply $\Gamma_{(0)} = \gamma^0$. For D2-branes, we have

$$\Gamma_{(0)}\eta_\pm = \gamma^0\epsilon^{i\bar{j}}\gamma_{i\bar{j}}\eta_\pm = i\gamma^0\gamma_{(6)}\eta_\pm \tag{5.5.17}$$

For D4-branes, we have

$$\Gamma_{(0)}\eta_\pm = \gamma^0\frac{1}{4}\epsilon^{i\bar{j}k\bar{l}}\gamma_{i\bar{j}k\bar{l}}\eta_\pm = -\gamma^0\eta_\pm \tag{5.5.18}$$

where we used the last column of 5.5.8. Finally for D6-branes, we have $\Gamma_{(0)} = -i\gamma^0\gamma_{(6)}$ using 5.5.3. These formulae can be summarized as $\Gamma_{(0)}\varepsilon = \gamma^0(i\gamma_{(6)})^{p/2}\varepsilon$.

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